

Acknowledgements

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The introduction shares some material with the author's article for the Proceedings of the 2014 ICM [[Hel14b](#)]. That article, in turn, is based in part on the informal note [[Hel13b](#)], which was published in Spanish translation [[Hel13a](#)] and in a revised French version [[Hel14a](#)]. Beyond the introduction, most of the book descends from the preprints [[Helb](#)], [[Hela](#)], [[Helc](#)], though much has changed significantly since then.

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The present work would most likely not have been possible without free and publicly available software: SageMath, PARI, Maxima, Sympy, gnuplot, ARB, GMP, MPFI, VNODE-LP, PROFIL / BIAS, and, of course, \LaTeX , Emacs, the gcc compiler and GNU/Linux in general. Some exploratory work was done in SageMath and Mathematica; SageMath was also used as an interface to Maxima and ARB, and to do minor calculations within the TeX files (via SageTeX). Floating-point calculations were always conducted rigorously by means of interval arithmetic. Most large interval-arithmetic computations used D. Platt's `int_double` package, based in part on Crlibm. Some other ones used ARB/FLINT, whether as a C library or, in the case of minor

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