THE GEOMETRY OF ALGEBRAIC VARIETIES
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Titles and Abstracts

Arnaud Beauville: Vector bundles on Fano threefolds and K3 surfaces.

Let X be a Fano threefold, and let $S \subset X$ be a smooth anticanonical K3 surface. Any moduli space $\mathcal{M}_S$ of simple vector bundles on S carries a holomorphic symplectic structure. Following an idea of Tyurin, I will show that in some cases, those vector bundles which come from X form a Lagrangian subvariety of $\mathcal{M}_S$. Most of the talk will be devoted to concrete examples of this situation.

Damian Brotbek: On a conjecture of Olivier Debarre.

In 2005, Olivier Debarre proved that general complete intersections of sufficiently ample hypersurfaces and of sufficiently high codimension in an abelian variety has ample cotangent bundle. He then conjectured that the same result holds in a projective space. In this talk we will discuss this conjecture and survey the different known results.

Fabrizio Catanese: Inoue type varieties and Hyperelliptic manifolds.

Together with Ingrid Bauer, inspired by a construction of M. Inoue, we defined an Inoue Type Variety to be an etale quotient $X = W/G$ of a hypersurface W in a projective classifying space Z.

For these varieties, the moduli spaces can be investigated by topological methods. A quite special case is the one of Hypersurfaces W in an Abelian variety A, whose moduli spaces were described in recent joint work with Yongnam Lee.

Another particular case is provided by hypersurfaces X in an Hyperelliptic variety, an etale quotient A/G (A an Abelian variety). Iitaka's conjecture asserts that any compact Kaehler manifold whose universal covering is $\mathbb{C}^n$ is a hyperelliptic manifold, and I proved with Demleitner and Claudon that every hyperelliptic manifold T/G deforms to an hyperelliptic variety A/G.

I shall discuss the moduli spaces of Hyperelliptic manifolds, their classification in small dimensions, the case where the group G is cyclic (Bagnera de Franchis Manifolds), and the Geometry of Hyperelliptic Varieties (some of this is joint work with Demleitner and with Corvaja).

For Hyperelliptic manifolds, their Kaehler Teichmueller space is easily described, and depends on the fundamental group and the Hodge type. By the results of Bieberbach, there is only a finite number of families in each dimension.

In dimension $n = 2$, Hyperelliptic manifolds were classified by Bagnera and De Franchis, and the group G is cyclic. We have a classification in dimension $n \leq 3$: there is only one family for $n = 3$ with group G non Abelian: the group is D_4.

One defines BdF manifolds as those Hyperelliptic manifolds with G cyclic. To study them, one needs to describe explicitly the action of a cyclic group on a complex torus. This can be done in an elementary way and is interesting also from the point of view of hypergeometric integrals.

Cinzia Casagrande: Special rational fibrations in Fano 4-folds.

Smooth, complex Fano 4-folds are not classified, and we still lack a good understanding of their general properties. We focus on Fano 4-folds with large second Betti number b_2, studied via birational geometry and the detailed analysis of their contractions and rational contractions (we recall that a contraction is a morphism with connected fibers onto a normal projective variety, and a rational contraction is given by a sequence of flips followed by a contraction). The main result that we want to present is the following: let X be a Fano 4-fold having a nonconstant rational contraction $X \dashrightarrow Y$ of fiber type. Then either $b_2(X)$ is at most 18, with equality only for a product of surfaces, or Y is $\mathbb{P}^1$ or $\mathbb{P}^2$. The proof is achieved by reducing to the case of "special" rational contractions of fiber type. We will explain this notion and give an idea of the techniques that are used.

Olivier Debarre: Gushel-Mukai varieties and their periods.

Gushel-Mukai varieties are defined as the intersection of the Grassmannian $\mathrm{Gr}(2,5)$ in its Plücker embedding, with a quadric and a linear space. They occur in dimension 6 (with a slighty modified construction), 5, 4, 3, 2 (where they are just K3 surfaces of degree 10), and 1 (where they are just genus 6 curves). Their theory parallels that of another important class of Fano varieties, cubic fourfolds, with many common features such as the presence of a canonically attached hyperkähler fourfold: the variety of lines for a cubic is replaced here with a double EPW sextic.

There is a big difference though: in dimension at least 3, GM varieties attached to a given EPW sextic form a family of positive dimension. However, we prove that the Hodge structure of any of these GM varieties can be reconstructed from that of the EPW sextic or of an associated surface of general type, depending on the parity of the dimension (for cubic fourfolds, the corresponding statement was proved in 1985 by Beauville and Donagi). This is joint work with Alexander Kuznetsov.

Tommaso de Fernex: Local structure of arc spaces.

Arc spaces are used in motivic integration and birational geometry, and their geometry has been studied since the 60's. I will talk about some questions regarding the local structure of arc spaces. I will present an approach based on a simple but useful formula for the sheaf of Kahler differentials of the arc space which leads to several properties of its local rings and their completions, and their relation to invariants of singularities. The talk is based on joint works with Roi Docampo and Christopher Chiu.

Stéphane Druel: On foliations with semi-positive anti-canonical bundle.

I will discuss the structure of (regular) foliations with semi-positive anti-canonical bundle on complex projective manifolds. I will also explain the (mostly conjectural) structure of foliations with numerically trivial canonical class.

Gavril Farkas: Quadric rank loci on moduli of curves and K3 surfaces.

Given two vector bundles E and F on a variety X and a morphism from $\mathrm{Sym}^2(E)$ to F, we compute the cohomology class of the locus in X where the kernel of this morphism contains a quadric of prescribed rank. Our formulas have many applications to moduli theory: (i) a simple proof of Borcherds' result that the Hodge class on the moduli space of polarized K3 surfaces of fixed genus is of Noether-Lefschetz type, (ii) an explicit canonical divisor on the Hurwitz space parametrizing degree k covers of the projective line from curves of genus $2k-1$, (iii) a closed formula for the Petri divisor on the moduli space of curves consisting of canonical curves which lie on a rank 3 quadric and (iv) myriads of effective divisors of small slope on M_g. Joint work with Rimanyi.

Daniel Huybrechts: Algebraic and arithmetic aspects of twistor spaces.

I will recall the well-known notion of twistor spaces for K3 surfaces (and Hyperkähler manifolds) and discuss some natural questions relating to the algebraic and arithmetic geometry of their fibres.

Alexander Kuznetsov: Rationality of Fano 3-folds over non-closed fields.

In the talk I will discuss rationality criteria for Fano 3-folds of geometric Picard number 1 over a non-closed field k of characteristic 0. Among these there are 8 types of geometrically rational varieties. We prove that in one of these cases any variety of this type is k-rational, in four cases the criterion of rationality is the existence of a k-rational point, and in the last three cases the criterion is the existence of a k-rational point and a k-rational curve of genus 0 and degree 1, 2, and 3 respectively. The last result is based on recent results of Benoist-Wittenberg. This is a joint work with Yuri Prokhorov.

Martí Lahoz: Stability conditions on non-commutative K3 surfaces.

The introduction by Bridgeland of a notion of stability for objects in triangulated categories has enabled the construction of moduli spaces on them. Not only this, but thanks to the great flexibility of this notion, it has allowed to study the birational geometry on these moduli spaces. Unfortunately, the main open problem is the existence of such stability conditions. Their construction for derived categories of smooth projective surfaces has had a tremendous impact for the study of the birational geometry of their moduli spaces. In particular, for the hyperkähler moduli spaces on K3 surfaces by the work of Bayer-Macrì.

In this talk, I will introduce the notion of stability conditions on triangulated categories and I will explain how to construct them on noncommutative K3 surfaces arising from cubic fourfolds. Finally, I will construct new locally complete polarized families of hyperkähler manifolds as moduli spaces of objects on these noncommutative K3 surfaces.

Robert Lazarsfeld: Cayley—Bacharach Theorems with Excess Vanishing.

A classical result usually attributed to Cayley and Bacharach asserts that if two plane curves of degrees c and d meet in cd points, then any curve of degree $(c + d - 3)$ passing through all but one of these points must also pass through the remaining one. In the late 1970s, Griffiths and Harris showed that this is a special case of a general result about zero-loci of sections of a vector bundle. Inspired by a recent paper of Mu-Lin Li, I will describe a generalization allowing for excess vanishing. Multiplier ideals enter the picture in a natural way. Time permitting, I will also explain how a result due to Tan and Viehweg leads to statements of Cayley-Bacharach type for determinantal loci. This is joint work with Lawrence Ein.

Emanuele Macrì: Hyperkähler fourfolds and Fano manifolds.

I will report on joint work in progress with Laure Flapan, Kieran O'Grady, and Giulia Saccà on how to naturally associate to a polarized hyperkähler fourfold, of divisibility 2, a Fano manifold of dimension depending on the degree of the polarization. The basic case is the correspondence between cubic fourfolds and their varieties of lines, with the Plücker polarization.

Laurent Manivel: Projections and jumps between Grassmannians.

Using geometric correspondences induced by projections and two-steps flag varieties, and a generalization of Orlov's projective bundle theorem, one can relate the Hodge structures and derived categories of subvarieties of different Grassmannians. This allows, among other things, to prove that the Peskine varieties admit three Hodge structures of K3 type, or to construct crepant categorical resolution of singularities of the Coble cubics (joint work with M. Bernardara and E. Fatighenti).

Mircea Mustaţă: Hodge filtration, minimal exponent, and local vanishing.

I will discuss a circle of ideas relating Saito's minimal exponent of a singularity, the Hodge filtration on the localization along a regular function, the V-filtration of Malgrange and Kashiwara, and local vanishing for differential forms with log poles. This is based on joint work with Mihnea Popa.

Kieran O'Grady: Natural sheaves on HK manifolds.

Holomorphic vector bundles on K3's, and more generally torsion free sheaves, play a key role in many non trivial results. We do not expect that torsion free sheaves on higher dimensional hyperkaehler (HK) manifolds behave as well as they do on K3 surfaces. I will describe a special class of sheaves on HK varieties, and I will motivate the expectation that they behave (almost) as well as sheaves on K3's.

John Ottem: Enriques fibrations with non-algebraic integral Hodge classes.

I will explain a construction of a certain pencil of Enriques surfaces with non-algebraic integral Hodge classes of non-torsion type. This gives the first example of a threefold with trivial Chow group of zero-cycles on which the integral Hodge conjecture fails. If time permits, I will explain an application to a classical question of Murre on the universality of the Abel-Jacobi maps in codimension three. This is joint work with Fumiaki Suzuki.