
Bibliography

- [AAR99] G. E. Andrews, R. Askey, and R. Roy. *Special functions*, volume 71. Cambridge: Cambridge University Press, 1999.
- [ACC⁺] C. Aldana, E. Carneiro, A. Chirre, H. Helfgott, and J. Mejía. Optimality for the two-parameter Selberg sieve. In preparation.
- [ADGR07] J. Avigad, K. Donnelly, D. Gray, and P. Raff. A formally verified proof of the prime number theorem. *ACM Trans. Comput. Log.*, 9(1):23, 2007.
- [Ahl78] L. V. Ahlfors. *Complex analysis: an introduction to the theory of analytic functions of one complex variable*. McGraw-Hill Book Co., New York, third edition, 1978.
- [AKS04] M. Agrawal, N. Kayal, and N. Saxena. PRIMES is in P. *Ann. of Math.* (2), 160(2):781–793, 2004.
- [Alt19] S. Zúñiga Alterman. *Smoothing and cancellation: the Barban-Vehov sieve made explicit*. PhD thesis, Université Paris 7, 2019.
- [AS64] M. Abramowitz and I. A. Stegun. *Handbook of mathematical functions with formulas, graphs, and mathematical tables*, volume 55 of *National Bureau of Standards Applied Mathematics Series*. For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C., 1964.
- [Atk49] F. V. Atkinson. The mean-value of the Riemann zeta function. *Acta Math.*, 81(1):353–376, 1949.
- [Axe10] A. Axer. Beitrag zur Kenntnis der zahlentheoretischen Funktionen $\mu(n)$ und $\lambda(n)$. *Prace Mat.-Fiz.*, 21:65–95, 1910.
- [Bac14] R. J. Backlund. Sur les zéros de la fonction $\zeta(s)$ de Riemann. *C. R. Acad. Sci., Paris*, 158:1979–1981, 1914.
- [Bac18] R. J. Backlund. Über die Nullstellen der Riemannschen Zetafunktion. *Acta Math.*, 41:345–375, 1918.
- [Bal78] R. Balasubramanian. An improvement on a theorem of Titchmarsh on

- the mean square of $|\zeta(1/2 + it)|$. *Proc. of the London Math. Soc.*, 3(3):540–576, 1978.
- [Bal12a] M. Balazard. Elementary remarks on the Möbius function. *Tr. Mat. Inst. Steklova*, 276:39–45, 2012.
 - [Bal12b] M. Balazard. Remarques élémentaires sur la fonction de Möbius. Detailed French version of [Bal12a]. Available at <https://hal.archives-ouvertes.fr/hal-00732694>, 2012.
 - [BBO10] J. Bertrand, P. Bertrand, and J.-P. Ovarlez. Mellin transform. In A. D. Poularikas, editor, *Transforms and applications handbook*. CRC Press, Boca Raton, FL, 2010.
 - [BD69] E. Bombieri and H. Davenport. On the large sieve method. In *Number Theory and Analysis (Papers in Honor of Edmund Landau)*, pages 9–22. Plenum, New York, 1969.
 - [BD06] A. Bonami and B. Demange. A survey on uncertainty principles related to quadratic forms. *Collect. Math.*, (Vol. Extra):1–36, 2006.
 - [BdR] M. Balazard and A. de Roton. Notes de lecture de l’article “Partial sums of the Möbius function” de Kannan Soundararajan. Unpublished. Available at <https://arxiv.org/abs/0810.3587>.
 - [Bet19] M. H. Betah. Explicit expression of a Barban & Vehov theorem. *Funct. Approximatio, Comment. Math.*, 60(2):177–193, 2019.
 - [Beu38] A. Beurling. Sur les intégrales de Fourier absolument convergentes et leur application à une transformation fonctionnelle. 9^{me} Congr. Math. Scand. 1938, 345–366 (1938), 1938.
 - [BM98] M. Berz and K. Makino. Verified integration of ODEs and flows using differential algebraic methods on high-order Taylor models. *Reliab. Comput.*, 4(4):361–369, 1998.
 - [BMOR18] M. A. Bennett, G. Martin, K. O’Bryant, and A. Rechnitzer. Explicit bounds for primes in arithmetic progressions. *Illinois J. Math.*, 62(1–4):427–532, 2018.
 - [Boc59] S. Bochner. *Lectures on Fourier integrals. With an author’s supplement on monotonic functions, Stieltjes integrals, and harmonic analysis.*, volume 42. Princeton University Press, Princeton, NJ, 1959.
 - [Bom74] E. Bombieri. *Le grand crible dans la théorie analytique des nombres*. Société Mathématique de France, Paris, 1974. Avec une sommaire en anglais, Astérisque, No. 18.
 - [Bom10] E. Bombieri. The classical theory of zeta and L -functions. *Milan J.*

- Math.*, 78(1):11–59, 2010.
- [Bom76] E. Bombieri. On twin almost primes. *Acta Arith.*, 28(2):177–193, 1975/76.
- [Boo06a] A. R. Booker. Artin’s conjecture, Turing’s method, and the Riemann hypothesis. *Experiment. Math.*, 15(4):385–407, 2006.
- [Boo06b] A. R. Booker. Turing and the Riemann hypothesis. *Notices Amer. Math. Soc.*, 53(10):1208–1211, 2006.
- [Bor49] J. L. Borges. Los teólogos. In *El aleph*. Losada, Buenos Aires, 1949.
- [Bor56] K. G. Borodzkin. On the problem of I. M. Vinogradov’s constant (in Russian). In *Proc. Third All-Union Math. Conf.*, volume 1, page 3. Izdat. Akad. Nauk SSSR, Moscow, 1956.
- [Bou99] J. Bourgain. On triples in arithmetic progression. *Geom. Funct. Anal.*, 9(5):968–984, 1999.
- [Bou17] J. Bourgain. Decoupling, exponential sums and the Riemann zeta function. *J. Amer. Math. Soc.*, 30(1):205–224, 2017.
- [BPCR06] S. Basu, R. Pollack, and M.-F. Coste-Roy. *Algorithms in real algebraic geometry*. Algorithms and computation in mathematics. Springer, Berlin, Heidelberg, New York, 2006.
- [But11] Y. Buttkewitz. Exponential sums over primes and the prime twin problem. *Acta Math. Hungar.*, 131(1-2):46–58, 2011.
- [BV68] M.B. Barban and P.P. Vekhov. On an extremal problem. *Trans. Mosc. Math. Soc.*, 18:91–99, 1968.
- [CD88] H. Cohen and F. Dress. Estimations numériques du reste de la fonction sommatoire relative aux entiers sans facteur carré. In *Colloque de Théorie Analytique des Nombres “Jean Coquet” (Marseille, 1985)*, volume 88 of *Publ. Math. Orsay*, pages 73–76. Univ. Paris XI, Orsay, 1988.
- [CDE07] H. Cohen, F. Dress, and M. El Marraki. Explicit estimates for summatory functions linked to the Möbius μ -function. *Funct. Approximatio, Comment. Math.*, 37:51–63, 2007.
- [CFDH⁺19] L. Cruz-Filipe, J. Davenport, H. Helfgott, J. Maynard, B. Poonen, and Ph. H. Tiep. Machine-assisted proofs. In *Proceedings of the International Congress of Mathematicians (ICM 2018)*. World Scientific, 2019.
- [CG04] Y. F. Cheng and S. W. Graham. Explicit estimates for the Riemann zeta

- function. *Rocky Mt. J. Math.*, 34(4):1261–1280, 2004.
- [Che73] J. R. Chen. On the representation of a larger even integer as the sum of a prime and the product of at most two primes. *Sci. Sinica*, 16:157–176, 1973.
- [Che85] J. R. Chen. On the estimation of some trigonometrical sums and their application. *Sci. Sinica Ser. A*, 28(5):449–458, 1985.
- [Che00] Y. Cheng. An explicit zero-free region for the Riemann zeta-function. *Rocky Mt. J. Math.*, pages 135–148, 2000.
- [Che12] S. Chevillard. The functions erf and erfc computed with arbitrary precision and explicit error bounds. *Information and Computation*, 216:72 – 95, 2012. Special Issue: 8th Conference on Real Numbers and Computers.
- [Chu37] N.G. Chudakov. On the Goldbach problem. *C. R. (Dokl.) Acad. Sci. URSS, n. Ser.*, 17:335–338, 1937.
- [Chu38] N.G. Chudakov. On the density of the set of even numbers which are not representable as the sum of two odd primes. *Izv. Akad. Nauk SSSR Ser. Mat.* 2, pages 25–40, 1938.
- [Chu47] N. G. Chudakov. *Introduction to the theory of Dirichlet L-functions*. OGIZ, Moscow-Leningrad, 1947. In Russian.
- [CK90] C. C. Chang and H. J. Keisler. *Model theory*, volume 73 of *Studies in Logic and the Foundations of Mathematics*. North-Holland Publishing Co., Amsterdam, third edition, 1990.
- [CKT70] W. J. Cody, A. Kathleen, and H. C. Thacher, jr. Chebyshev approximations for Dawson’s integral. *Math. Comput.*, 24:171–178, 1970.
- [Coh] H. Cohen. High precision computation of Hardy-Littlewood constants. Preprint.
- [Coh07] H. Cohen. *Number theory. Vol. II. Analytic and modern tools*, volume 240 of *Graduate Texts in Mathematics*. Springer, New York, 2007.
- [Col75] G. E. Collins. Quantifier elimination for real closed fields by cylindrical algebraic decomposition. In *Automata theory and formal languages (Second GI Conf., Kaiserslautern, 1975)*, pages 134–183. Lecture Notes in Comput. Sci., Vol. 33. Springer, Berlin, 1975.
- [CR08] S. Chevillard and N. Revol. Computation of the error function erf in arbitrary precision with correct rounding. In J. D. Bruguera and M. Daumas, editors, *RNC 8 Proceedings, 8th Conference on Real Numbers and Computers*, pages 27–36, July 2008.

- [CV10a] E. Carneiro and J. D. Vaaler. Some extremal functions in Fourier analysis. II. *Trans. Am. Math. Soc.*, 362(11):5803–5843, 2010.
- [CV10b] E. Carneiro and J. D. Vaaler. Some extremal functions in Fourier analysis. III. *Constr. Approx.*, 31(2):259–288, 2010.
- [CW89] J. R. Chen and T. Z. Wang. On the Goldbach problem. *Acta Math. Sinica*, 32(5):702–718, 1989.
- [CW96] J. R. Chen and T. Z. Wang. The Goldbach problem for odd numbers. *Acta Math. Sinica (Chin. Ser.)*, 39(2):169–174, 1996.
- [Dab96] H. Daboussi. Effective estimates of exponential sums over primes. In *Analytic number theory, Vol. 1 (Allerton Park, IL, 1995)*, volume 138 of *Progr. Math.*, pages 231–244. Birkhäuser Boston, Boston, MA, 1996.
- [Dab01] H. Daboussi. Brun’s fundamental lemma and exponential sums over primes. *J. Number Theory*, 90(1):1–18, 2001.
- [Dah52] G. Dahlquist. On the analytic continuation of Eulerian products. *Ark. Mat.*, 1:533–554, 1952.
- [Dav67] H. Davenport. *Multiplicative number theory*. Markham Publishing Co., Chicago, Ill., 1967. Lectures given at the University of Michigan, Winter Term.
- [dB81] N. G. de Bruijn. *Asymptotic methods in analysis*. Dover Publications Inc., New York, third edition, 1981.
- [DE93] F. Dress and M. El Marraki. Fonction sommatoire de la fonction de Möbius. II: Majorations asymptotiques élémentaires. *Exp. Math.*, 2(2):99–112, 1993.
- [Des08] R. Descartes. Œuvres de Descartes publiées par Charles Adam et Paul Tannery sous les auspices du Ministère de l’Instruction publique. Physico-mathematica. Compendium musicae. Regulae ad directionem ingenii. Recherche de la vérité. Supplément à la correspondance. X. Paris: Léopold Cerf. IV u. 691 S. 4°, 1908.
- [Des77] J.-M. Deshouillers. Sur la constante de Šnirel'man. In *Séminaire Delange-Pisot-Poitou, 17e année: (1975/76), Théorie des nombres: Fac. 2, Exp. No. G16*, page 6. Secrétariat Math., Paris, 1977.
- [DEtRZ97] J.-M. Deshouillers, G. Effinger, H. te Riele, and D. Zinoviev. A complete Vinogradov 3-primes theorem under the Riemann hypothesis. *Electron. Res. Announc. Amer. Math. Soc.*, 3:99–104, 1997.
- [DHA19] D. Dona, H. A. Helfgott, and S. Zúñiga Alerman. Explicit L^2 bounds

- for the Riemann ζ function. Preprint. Available as <https://arxiv.org/abs/1906.01097.pdf>, 2019.
- [Dic66] L. E. Dickson. *History of the theory of numbers. Vol. I: Divisibility and primality*. Chelsea Publishing Co., New York, 1966.
 - [DIT83] F. Dress, H. Iwaniec, and G. Tenenbaum. Sur une somme liée à la fonction de Möbius. *J. Reine Angew. Math.*, 340:53–58, 1983.
 - [dlBDT] R. de la Bretèche, F. Dress, and G. Tenenbaum. Remarques sur une somme liée à la fonction de Möbius. Preprint. Available as <https://arxiv.org/abs/1902.09956>.
 - [DLDDDD⁺10] C. Daramy-Loirat, F. De Dinechin, D. Defour, M. Gallet, N. Gast, and Ch. Lauter. Crlibm, March 2010. version 1.0beta4.
 - [DR96] M. Deléglise and J. Rivat. Computing the summation of the Möbius function. *Experiment. Math.*, 5(4):291–295, 1996.
 - [DR01] H. Daboussi and J. Rivat. Explicit upper bounds for exponential sums over primes. *Math. Comp.*, 70(233):431–447 (electronic), 2001.
 - [Dre93] F. Dress. Fonction sommatoire de la fonction de Möbius. I. Majorations expérimentales. *Experiment. Math.*, 2(2):89–98, 1993.
 - [DS70] H. G. Diamond and J. Steinig. An elementary proof of the prime number theorem with a remainder term. *Invent. Math.*, 11:199–258, 1970.
 - [Dus98] P. Dusart. *Autour de la fonction qui compte le nombre de nombres premiers*. PhD thesis, Université de Limoges, 1998.
 - [Dus16] P. Dusart. Estimates of ψ , θ for large values of x without the Riemann hypothesis. *Math. Comput.*, 85(298):875–888, 2016.
 - [Ebe19] M. Eberl. Nine Chapters of Analytic Number Theory in Isabelle/HOL. In John Harrison, John O’Leary, and Andrew Tolmach, editors, *10th International Conference on Interactive Theorem Proving (ITP 2019)*, volume 141 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 16:1–16:19, Dagstuhl, Germany, 2019. Schloss Dagstuhl–Leibniz-Zentrum für Informatik.
 - [Edw74] H. M. Edwards. *Riemann’s zeta function*. Academic Press [A subsidiary of Harcourt Brace Jovanovich, Publishers], New York-London, 1974. Pure and Applied Mathematics, Vol. 58.
 - [Eff99] G. Effinger. Some numerical implications of the Hardy and Littlewood analysis of the 3-primes problem. *Ramanujan J.*, 3(3):239–280, 1999.
 - [EM95] M. El Marraki. Fonction sommatoire de la fonction de Möbius. III. Ma-

- jorations asymptotiques effectives fortes. *J. Théor. Nombres Bordeaux*, 7(2):407–433, 1995.
- [EM96] M. El Marraki. Majorations de la fonction sommatoire de la fonction $\frac{\mu(n)}{n}$. Univ. Bordeaux 1, preprint (96-8), 1996.
- [Est28] T. Estermann. On certain functions represented by Dirichlet's series. *Proc. Lond. Math. Soc.* (2), 27:435–448, 1928.
- [Est37] T. Estermann. On Goldbach's Problem : Proof that Almost all Even Positive Integers are Sums of Two Primes. *Proc. London Math. Soc.*, S2-44(4):307–314, 1937.
- [FHL⁺07] L. Fousse, G. Hanrot, V. Lefèvre, P. Péllissier, and P. Zimmermann. Mpfr: A multiple-precision binary floating-point library with correct rounding. *ACM Trans. Math. Softw.*, 33:000000818, 2007.
- [FI98] J. Friedlander and H. Iwaniec. Asymptotic sieve for primes. *Ann. of Math.* (2), 148(3):1041–1065, 1998.
- [FI10] J. Friedlander and H. Iwaniec. *Opera de cribro*, volume 57 of *American Mathematical Society Colloquium Publications*. American Mathematical Society, Providence, RI, 2010.
- [FK15] L. Faber and H. Kadiri. New bounds for $\psi(x)$. *Math. Comput.*, 84(293):1339–1357, 2015.
- [FK18] L. Faber and H. Kadiri. Corrigendum to New bounds for $\psi(x)$. *Math. Comp.*, 87(311):1451–1455, 2018.
- [FLM14] K. Ford, F. Luca, and P. Moree. Values of the Euler φ -function not divisible by a given odd prime, and the distribution of Euler-Kronecker constants for cyclotomic fields. *Math. Comput.*, 83(287):1447–1476, 2014.
- [For02a] K. Ford. Vinogradov's integral and bounds for the Riemann zeta function. *Proc. London Math. Soc.* (3), 85(3):565–633, 2002.
- [For02b] K. Ford. Zero-free regions for the Riemann zeta function. In *Number theory for the millennium. II (Urbana, IL, 2000)*, volume 2556, 2002.
- [FS13] D. A. Frolenkov and K. Soundararajan. A generalization of the pôlya-vinogradov inequality. *The Ramanujan Journal*, 31(3):271–279, Aug 2013.
- [Gal00] W. F. Galway. Dissecting a sieve to cut its need for space. In *Algorithmic number theory (Leiden, 2000)*, volume 1838 of *Lecture Notes in Comput. Sci.*, pages 297–312. Springer, Berlin, 2000.

- [GD04] X. Gourdon and P. Demichel. The first 10^{13} zeros of the Riemann zeta function, and zeros computation at very large height. <http://numbers.computation.free.fr/Constants/Miscellaneous/zetazeros1e13-1e24.pdf>, 2004.
- [Gol85] D. Goldfeld. Gauss's class number problem for imaginary quadratic fields. *Bull. Amer. Math. Soc. (N.S.)*, 13(1):23–37, 1985.
- [Gol06] D. Goldfeld. *Automorphic forms and L-functions for the group $GL(n, \mathbb{R})$. With an appendix by Kevin A. Broughan*. Cambridge: Cambridge University Press, 2006.
- [Gon89] S. M. Gonek. On negative moments of the Riemann zeta-function. *Mathematika*, 36(1):71–88, 1989.
- [GPY09] D. A. Goldston, J. Pintz, and C. Y. Yıldırım. Primes in tuples. I. *Ann. of Math.* (2), 170(2):819–862, 2009.
- [GR94] I. S. Gradshteyn and I. M. Ryzhik. *Table of integrals, series, and products*. Academic Press, Inc., Boston, MA, fifth edition, 1994. Translation edited and with a preface by Alan Jeffrey.
- [GR96] A. Granville and O. Ramaré. Explicit bounds on exponential sums and the scarcity of squarefree binomial coefficients. *Mathematika*, 43(1):73–107, 1996.
- [Gra84] J. P. Gram. Undersøgelser angående Mængden af Primtal under en given Grænse (avec un résumé en français). *Kjobenhavn. Skrift.*, II(6):185–308, 1884.
- [Gra03] J.-P. Gram. Note sur les zéros de la fonction $\zeta(s)$ de Riemann. *Acta Math.*, 27:289–304, 1903.
- [Gra78] S. Graham. An asymptotic estimate related to Selberg's sieve. *J. Number Theory*, 10:83–94, 1978.
- [GS68] I. M. Gel'fand and G. E. Shilov. *Generalized functions. Vol. 2. Spaces of fundamental and generalized functions*. Translated from the Russian by Morris D. Friedman, Amiel Feinstein and Christian P. Peltzer. Academic Press, New York-London, 1968.
- [GST04] A. Gil, J. Segura, and N. M. Temme. Integral representations for computing real parabolic cylinder functions. *Numerische Mathematik*, 98(1):105–134, 2004.
- [GST06] A. Gil, J. Segura, and N. M. Temme. Computing the real parabolic cylinder functions $U(a, x)$, $V(a, x)$. *ACM Trans. Math. Softw.*, 32(1):70–101, 2006.

- [Gui42] A. P. Guinand. Summation formulae and self-reciprocal functions. III. *Q. J. Math., Oxf. Ser.*, 13:30–39, 1942.
- [GV81] S.W. Graham and J. D. Vaaler. A class of extremal functions for the Fourier transform. *Trans. Am. Math. Soc.*, 265:283–302, 1981.
- [GY02] D. A. Goldston and C. Y. Yıldırım. Higher correlations of divisor sums related to primes III: k -correlations. Preprint. Available as <https://arxiv.org/abs/0209102>, 2002.
- [Hal74] P. R. Halmos. *Measure theory*, volume 18. Springer, New York, NY, 1974.
- [Har33] G.H. Hardy. A theorem concerning Fourier transforms. *J. Lond. Math. Soc.*, 8:227–231, 1933.
- [Har66] G. H. Hardy. *Collected papers of G. H. Hardy (Including Joint papers with J. E. Littlewood and others). Vol. I.* Edited by a committee appointed by the London Mathematical Society. Clarendon Press, Oxford, 1966.
- [Har09] J. Harrison. Formalizing an analytic proof of the prime number theorem. *J. Autom. Reasoning*, 43(3):243–261, 2009.
- [HB79] D. R. Heath-Brown. The fourth power moment of the Riemann zeta function. *Proc. London Math. Soc. (3)*, 38(3):385–422, 1979.
- [HB85] D. R. Heath-Brown. The ternary Goldbach problem. *Rev. Mat. Iberoamericana*, 1(1):45–59, 1985.
- [HB11] H. Hong and Ch. W. Brown. QEPCAD B – Quantifier elimination by partial cylindrical algebraic decomposition, May 2011. version 1.62.
- [Hea92] D. R. Heath-Brown. Zero-free regions for Dirichlet L -functions and the least prime in an arithmetic progression. *Proc. Lond. Math. Soc.*, III. Ser. 64, No. 2, 265–338 (1992),, 1992.
- [Hej89] D. A. Hejhal. On the distribution of $\log |\zeta'(\frac{1}{2} + it)|$. In *Number theory, trace formulas and discrete groups (Oslo, 1987)*, pages 343–370. Academic Press, Boston, MA, 1989.
- [Hela] H. A. Helfgott. Major arcs for Goldbach’s problem. Preprint. Available at <https://arxiv.org/abs/1305.2897>.
- [Helb] H. A. Helfgott. Minor arcs for Goldbach’s problem. Preprint. Available as <https://arxiv.org/abs/1205.5252>.
- [Helc] H. A. Helfgott. The Ternary Goldbach Conjecture is true. Preprint. Available as <https://arxiv.org/abs/1312.7748>.

- [Hel13a] H. Helfgott. La conjetura débil de Goldbach. *Gac. R. Soc. Mat. Esp.*, 16(4), 2013.
- [Hel13b] H. A. Helfgott. The ternary Goldbach conjecture, 2013. Available at <http://valuevar.wordpress.com/2013/07/02/the-ternary-goldbach-conjecture/>.
- [Hel14a] H. A. Helfgott. La conjecture de Goldbach ternaire. *Gaz. Math.*, (140):5–18, 2014. Translated by Margaret Bilu, revised by the author.
- [Hel14b] H. A. Helfgott. The ternary Goldbach problem. In *Proceedings of the International Congress of Mathematicians (ICM 2014), Seoul, Korea, August 13–21, 2014. Vol. II: Invited lectures*, pages 391–418. Seoul: KM Kyung Moon Sa, 2014.
- [Hel20] Harald Andrés Helfgott. An improved sieve of Eratosthenes. *Math. Comp.*, 89(321):333–350, 2020.
- [Hia16] Gh. A. Hiary. An alternative to Riemann-Siegel type formulas. *Math. Comp.*, 85(298):1017–1032, 2016.
- [HL17] G. H. Hardy and J. E. Littlewood. Contributions to the theory of the Riemann Zeta-function and the theory of the distribution of primes. *Acta Math.*, 41:119–196, 1917.
- [HL22a] G. H. Hardy and J. E. Littlewood. Some problems of ‘Partitio numerorum’; III: On the expression of a number as a sum of primes. *Acta Math.*, 44(1):1–70, 1922.
- [HL22b] G. H. Hardy and J. E. Littlewood. The approximate functional equation in the theory of the zeta-function, with applications to the divisor-problems of Dirichlet and Piltz. *Proc. Lond. Math. Soc. (2)*, 21:39–74, 1922.
- [HP13] H. A. Helfgott and D. J. Platt. Numerical verification of the ternary Goldbach conjecture up to $8.875 \cdot 10^{30}$. *Exp. Math.*, 22(4):406–409, 2013.
- [HR00] G. H. Hardy and S. Ramanujan. Asymptotic formulæ in combinatory analysis [Proc. London Math. Soc. (2) 17 (1918), 75–115]. In *Collected papers of Srinivasa Ramanujan*, pages 276–309. AMS Chelsea Publ., Providence, RI, 2000.
- [HS17] A. J. Harper and K. Soundararajan. Lower bounds for the variance of sequences in arithmetic progressions: primes and divisor functions. *Q. J. Math.*, 68(1):97–123, 2017.
- [Hur18] G. Hurst. Computations of the Mertens function and improved bounds on the Mertens conjecture. *Mathematics of Computation*,

- 87(310):1013–1028, 2018.
- [Hut25] J. I. Hutchinson. On the roots of the Riemann zeta function. *Trans. Am. Math. Soc.*, 27:49–60, 1925.
- [Hux68] M.N. Huxley. The large sieve inequality for algebraic number fields. *Mathematika*, 15:178–187, 1968.
- [Hux72] M. N. Huxley. Irregularity in sifted sequences. *J. Number Theory*, 4:437–454, 1972.
- [HW79] G. H. Hardy and E. M. Wright. An introduction to the theory of numbers. 5th ed. Oxford etc.: Oxford at the Clarendon Press. XVI, 426 p. hbk., 1979.
- [IEE08] IEEE Standard for floating-point arithmetic. *IEEE Std 754-2008*, pages 1–70, Aug 2008.
- [IK04] H. Iwaniec and E. Kowalski. *Analytic number theory*, volume 53 of *American Mathematical Society Colloquium Publications*. American Mathematical Society, Providence, RI, 2004.
- [IMS09] Y. Ihara, V. K. Murty, and M. Shimura. On the logarithmic derivatives of Dirichlet L -functions at $s = 1$. *Acta Arith.*, 137(3):253–276, 2009.
- [Inc19] OEIS Foundation Inc. The on-line encyclopedia of integer sequences. <http://oeis.org>, 2019.
- [Ing27] A. E. Ingham. Mean-value theorems in the theory of the Riemann Zeta-function. *Proc. Lond. Math. Soc. (2)*, 27:273–300, 1927.
- [Iwa97] H. Iwaniec. *Topics in classical automorphic forms*. Providence, RI: American Mathematical Society, 1997.
- [Iwa14] H. Iwaniec. *Lectures on the Riemann zeta function*, volume 62 of *University Lecture Series*. American Mathematical Society, Providence, RI, 2014.
- [Joh13] F. Johansson. Arb: a C library for ball arithmetic. *ACM Communications in Computer Algebra*, 47(4):166–169, 2013.
- [Joh16] F. Johansson. Taking the error out of the error function, 2016. Available at <http://fredrikj.net/blog/2016/03/taking-the-error-out-of-the-error-function/>.
- [Joh18] F. Johansson. Numerical integration in arbitrary-precision ball arithmetic. In *International Congress on Mathematical Software*, pages 255–263. Springer, 2018.
- [Joh19] Fredrik Johansson. Computing hypergeometric functions rigorously.

- ACM Transactions on Mathematical Software (TOMS)*, 45(3):30, 2019.
- [Jut79a] M. Jutila. Corrigendum: “On a problem of Barban and Vehov”. *Mathematika*, 26(2):306 (1980), 1979.
- [Jut79b] M. Jutila. On a problem of Barban and Vehov. *Mathematika*, 26(1):62–71, 1979.
- [Kad02] H Kadiri. *Une région explicite sans zéro pour les fonctions L de Dirichlet Ph. D.* PhD thesis, Université de Lille I, 2002.
- [Kad05] H. Kadiri. Une région explicite sans zéros pour la fonction ζ de Riemann. *Acta Arith.*, 117(4):303–339, 2005.
- [Kad13] H. Kadiri. A zero density result for the Riemann zeta function. *Acta Arith.*, 160(2):185–200, 2013.
- [Kad18] H. Kadiri. Explicit zero-free regions for Dirichlet L -functions. *Mathematika*, 64(2):445–474, 2018.
- [Kan89] S. Kanemitsu. On evaluation of certain limits in closed form. Théorie des nombres, C. R. Conf. Int., Québec/Can. 1987, 459–474 (1989), 1989.
- [Kar93] A. A. Karatsuba. *Basic analytic number theory*. Springer-Verlag, Berlin, 1993. Translated from the second (1983) Russian edition and with a preface by Melvyn B. Nathanson.
- [Khi64] A. Ya. Khinchin. Continued fractions. Chicago and London: The University of Chicago Press. xi, 95 pp. (1964)., 1964.
- [KL14] H. Kadiri and A. Lumley. Short effective intervals containing primes. *Integers*, 14:Paper No. A61, 18, 2014.
- [KN12] H. Kadiri and N. Ng. Explicit zero density theorems for Dedekind zeta functions. *J. Number Theory*, 132(4):748–775, 2012.
- [Knü99] O. Knüppel. PROFIL/BIAS, February 1999. version 2.
- [Kor58] N. M. Korobov. Estimates of trigonometric sums and their applications. *Uspehi Mat. Nauk*, 13(4 (82)):185–192, 1958.
- [Kuz] E. Kuznetsov. Computing the Mertens function on a GPU. Preprint. Available as <https://arxiv.org/abs/1108.0135>.
- [KvdL04] T. Kotnik and J. van de Lune. On the order of the Mertens function. *Experiment. Math.*, 13(4):473–481, 2004.
- [Lam08] B. Lambov. Interval arithmetic using SSE-2. In *Reliable Implementation of Real Number Algorithms: Theory and Practice. Interna-*

- tional Seminar Dagstuhl Castle, Germany, January 8–13, 2006*, volume 5045 of *Lecture Notes in Computer Science*, pages 102–113. Springer, Berlin, 2008.
- [Lan06] E. Landau. Über den Zusammenhang einiger neuerer Sätze der analytischen Zahlentheorie. *Wien. Ber.*, 115:589–632, 1906.
 - [Lan09a] E. Landau. Handbuch der Lehre von der Verteilung der Primzahlen. Erster Band. Leipzig u. Berlin: B. G. Teubner. X + 564 S., 1909.
 - [Lan09b] E. Landau. Handbuch der Lehre von der Verteilung der Primzahlen. Zweiter Band. Leipzig u. Berlin: B. G. Teubner. IX u. S. 567–961, 1909.
 - [Lan18a] E. Landau. Über die Klassenzahl imaginär-quadratischer Zahlkörper. *Nachr. Ges. Wiss. Göttingen, Math.-Phys. Kl.*, 1918:285–295, 1918.
 - [Lan18b] E. Landau. Über einige ältere Vermutungen und Behauptungen in der Primzahltheorie. I, II. *Math. Z.*, 1:1–24, 213–219, 1918.
 - [Lan19] E. Landau. Zur Theorie der Heckeschen Zetafunktionen, welche komplexen Charakteren entsprechen. *Math. Z.*, 4:152–162, 1919.
 - [Lan35] E. Landau. Bemerkungen zum Heilbronnschen Satz. *Acta Arith.*, 1:1–18, 1935.
 - [Lan94] S. Lang. *Algebraic number theory*. 2nd ed., volume 110. New York: Springer-Verlag, 2nd ed. edition, 1994.
 - [Lan19] A. Languasco. A note on the computation of the Euler-Kronecker constants for prime cyclotomic fields. Preprint. Available as <https://arxiv.org/abs/1903.05487.pdf>, 2019.
 - [Leh56] D. H. Lehmer. Extended computation of the Riemann zeta-function. *Mathematika*, 3:102–108, 1956.
 - [Leh66] R. Sh. Lehman. On the difference $\pi(x) - \text{li}(x)$. *Acta Arith.*, 11:397–410, 1966.
 - [Leh70] R. S. Lehman. On the distribution of zeros of the Riemann zeta-function. *Proc. Lond. Math. Soc. (3)*, 20:303–320, 1970.
 - [Lin41] U. V. Linnik. The large sieve. *C. R. (Doklady) Acad. Sci. URSS (N.S.)*, 30:292–294, 1941.
 - [Lin46] U. V. Linnik. A new proof of the Goldbach-Vinogradow theorem. *Rec. Math. [Mat. Sbornik] N.S.*, 19 (61):3–8, 1946.
 - [Lit24] J. E. Littlewood. On the zeros of the Riemann Zeta-function. *Proc. Camb. Philos. Soc.*, 22:295–318, 1924.

- [Lit28] J. E. Littlewood. On the class-number of the corpus $P(\sqrt{-k})$. *Proc. Lond. Math. Soc.* (2), 27:358–372, 1928.
- [Lit06] F. Littmann. One-sided approximation by entire functions. *J. Approx. Theory*, 141(1):1–7, 2006.
- [LLS15] Y. Lamzouri, X. Li, and K. Soundararajan. Conditional bounds for the least quadratic non-residue and related problems. *Math. Comp.*, 84(295):2391–2412, 2015.
- [LO87] J. C. Lagarias and A. M. Odlyzko. Computing $\pi(x)$: an analytic method. *J. Algorithms*, 8(2):173–191, 1987.
- [Luc] Lucia. Answer to question $|L'(1, \chi)/L(1, \chi)|$. MathOverflow. <https://mathoverflow.net/q/337456> (version: 2019-08-02).
- [LW02] M.-Ch. Liu and T. Wang. On the Vinogradov bound in the three primes Goldbach conjecture. *Acta Arith.*, 105(2):133–175, 2002.
- [Mac94] R.A. MacLeod. A curious identity for the Möbius function. *Util. Math.*, 46:91–95, 1994.
- [Mar41] K. K. Mardzhanishvili. On the proof of the Goldbach-Vinogradov theorem (in Russian). *C. R. (Doklady) Acad. Sci. URSS (N.S.)*, 30(8):681–684, 1941.
- [May15] J. Maynard. Small gaps between primes. *Ann. of Math.* (2), 181(1):383–413, 2015.
- [McC74] J. H. McCabe. A continued fraction expansion, with a truncation error estimate, for Dawson’s integral. *Math. Comput.*, 28(127):811–816, July 1974.
- [McC84a] K. S. McCurley. Explicit estimates for the error term in the prime number theorem for arithmetic progressions. *Math. Comp.*, 42(165):265–285, 1984.
- [McC84b] K. S. McCurley. Explicit zero-free regions for Dirichlet L -functions. *J. Number Theory*, 19(1):7–32, 1984.
- [Mei54] E. Meissel. Observationes quaedam in theoria numerorum. *J. reine angew. Math.*, 48:301–316, 1854.
- [MM93] K. Matsumoto and T. Meurman. The mean square of the Riemann zeta-function in the critical strip III. *Acta Arith.*, 64:357–382, 1993.
- [Mon68] H. L. Montgomery. A note on the large sieve. *J. London Math. Soc.*, 43:93–98, 1968.

- [Mon71] H. L. Montgomery. *Topics in multiplicative number theory*. Lecture Notes in Mathematics, Vol. 227. Springer-Verlag, Berlin, 1971.
- [Mor00] P. Moree. Approximation of singular series and automata. *Manuscr. Math.*, 101(3):385–399, 2000.
- [Mor05] P. Moree. The formal series Witt transform. *Discrete Math.*, 295(1–3):143–160, 2005.
- [Mot74] Y. Motohashi. A problem in the theory of sieves. *Kokyuroku RIMS Kyoto Univ*, 222:9–50, 1974. In Japanese.
- [Mot76] Y. Motohashi. On a density theorem of Linnik. *Proc. Japan Acad.*, 51:815–817, 1976.
- [Mot78] Y. Motohashi. Primes in arithmetic progressions. *Invent. Math.*, 44(2):163–178, 1978.
- [Mot83] Y. Motohashi. *Lectures on sieve methods and prime number theory*, volume 72 of *Tata Institute of Fundamental Research Lectures on Mathematics and Physics*. Published for the Tata Institute of Fundamental Research, Bombay; by Springer-Verlag, Berlin, 1983.
- [Mot04] Y. Motohashi. A multiple sum involving the Möbius function. *Publ. Inst. Math., Nouv. Sér.*, 76:31–39, 2004.
- [MRS] P. Moree, O. Ramaré, and A. Sedunova. Arithmetical functions: three walks to higher ground. Book in progress.
- [MT15] M. J. Mossinghoff and T. S. Trudgian. Nonnegative trigonometric polynomials and a zero-free region for the Riemann zeta-function. *J. Number Theory*, 157:329–349, 2015.
- [MV73] H. L. Montgomery and R. C. Vaughan. The large sieve. *Mathematika*, 20:119–134, 1973.
- [MV74] H. L. Montgomery and R. C. Vaughan. Hilbert’s inequality. *J. London Math. Soc.* (2), 8:73–82, 1974.
- [MV07] H. L. Montgomery and R. C. Vaughan. *Multiplicative number theory. I. Classical theory*, volume 97 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2007.
- [Naz93] F. L. Nazarov. Local estimates for exponential polynomials and their applications to inequalities of the uncertainty principle type. *Algebra i Analiz*, 5(4):3–66, 1993.
- [Ned06] N. S. Nedialkov. VNODE-LP: a validated solver for initial value problems in ordinary differential equations, July 2006. version 0.3.

- [Nie97] O. A. Nielsen. *Introduction to integration and measure theory*. New York, NY: John Wiley & Sons, 1997.
- [OeS] T. Oliveira e Silva. Fast implementation of the segmented sieve of Eratosthenes. http://sweet.ua.pt/tos/software/prime_sieve.html. Accessed: 2016-6-22.
- [Oes88] J. Oesterlé. Le problème de Gauss sur le nombre de classes. *Enseign. Math.* (2), 34(1-2):43–67, 1988.
- [OeSHP14] T. Oliveira e Silva, S. Herzog, and S. Pardi. Empirical verification of the even Goldbach conjecture, and computation of prime gaps, up to $4 \cdot 10^{18}$. *Math. Comp.*, 83:2033–2060, 2014.
- [OLBC10] F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and Ch. W. Clark, editors. *NIST handbook of mathematical functions*. U.S. Department of Commerce National Institute of Standards and Technology, Washington, DC, 2010. With 1 CD-ROM (Windows, Macintosh and UNIX).
- [Olv58] F. W. J. Olver. Uniform asymptotic expansions of solutions of linear second-order differential equations for large values of a parameter. *Philos. Trans. Roy. Soc. London. Ser. A*, 250:479–517, 1958.
- [Olv59] F. W. J. Olver. Uniform asymptotic expansions for Weber parabolic cylinder functions of large orders. *J. Res. Nat. Bur. Standards Sect. B*, 63B:131–169, 1959.
- [Olv61] F. W. J. Olver. Two inequalities for parabolic cylinder functions. *Proc. Cambridge Philos. Soc.*, 57:811–822, 1961.
- [Olv65] F. W. J. Olver. On the asymptotic solution of second-order differential equations having an irregular singularity of rank one, with an application to Whittaker functions. *J. Soc. Indust. Appl. Math. Ser. B Numer. Anal.*, 2:225–243, 1965.
- [Olv70] F. W. J. Olver. Why steepest descents? *SIAM Rev.*, 12:228–247, 1970.
- [Olv74] F. W. J. Olver. *Asymptotics and special functions*. Academic Press [A subsidiary of Harcourt Brace Jovanovich, Publishers], New York-London, 1974. Computer Science and Applied Mathematics.
- [OS88] A.M. Odlyzko and A. Schönhage. Fast algorithms for multiple evaluations of the Riemann zeta function. *Trans. Am. Math. Soc.*, 309(2):797–809, 1988.
- [OtR85] A. M. Odlyzko and H. J. J. te Riele. Disproof of the Mertens conjecture. *J. Reine Angew. Math.*, 357:138–160, 1985.
- [Pag35] A. Page. On the number of primes in an arithmetic progression. *Proc.*

- Lond. Math. Soc.* (2), 39:116–141, 1935.
- [Pea09] J. W. Pearson. Computation of hypergeometric functions. Master's thesis, University of Oxford, 2009. Available at http://people.maths.ox.ac.uk/~porterm/research/pearson_final.pdf.
- [Pla11] D. Platt. *Computing degree 1 L-functions rigorously*. PhD thesis, Bristol University, 2011.
- [Pla15] D. J. Platt. Computing $\pi(x)$ analytically. *Math. Comp.*, 84(293):1521–1535, 2015.
- [Pla16] D. J. Platt. Numerical computations concerning the GRH. *Math. Comput.*, 85(302):3009–3027, 2016.
- [Poi77] G. Poitou. Minorations de discriminants (d'après A. M. Odlyzko). Sémin. Bourbaki 1975/76, 28ème Année, Exposés 471-488, Lect. Notes Math. 567, Exp. No. 479, 136-153 (1977), 1977.
- [Pol18] G. Polya. Über die Verteilung der quadratischen Reste und Nichtreste. *Göttingen Nachrichten*, 167:21–29, 1918.
- [Pom11] C. Pomerance. Remarks on the Pólya-Vinogradov inequality. *Integers*, 11(4):531–542, a19, 2011.
- [POP17] J. W. Pearson, Sh. Olver, and M. A. Porter. Numerical methods for the computation of the confluent and Gauss hypergeometric functions. *Numerical Algorithms*, 74(3):821–866, 2017.
- [Pre84] E. Preissmann. Sur une inégalité de Montgomery-Vaughan. *Enseign. Math.* (2), 30:95–113, 1984.
- [Pro78] F. Proth. Théorèmes sur les nombres premiers. *Comptes Rendus des Séances de l'Académie des Sciences, Paris*, 87:926, 1878.
- [PT16] D. J. Platt and T. S. Trudgian. On the first sign change of $\theta(x) - x$. *Math. Comput.*, 85(299):1539–1547, 2016.
- [PTM14] Project Team MARELL. Mathematical Reasoning and Software: activity report 2014. Technical report, Sophia Antipolis - Méditerranée, 2014. Theme: Proofs and Verification.
- [RA17] O. Ramaré and P. Akhilesh. Explicit averages of non-negative multiplicative functions: going beyond the main term. *Colloq. Math.*, 147(2):275–313, 2017.
- [Rad60] H. Rademacher. On the Phragmén-Lindelöf theorem and some applications. *Math. Z.*, 72:192–204, 1959/1960.

- [Rama] O. Ramaré. État des lieux. Unpublished. Available as <http://iml.univ-mrs.fr/~ramare/Maths/ExplicitJNTB.pdf>.
- [Ramb] O. Ramaré. Explicit average orders: news and problems. Preprint.
- [Ram95] O. Ramaré. On Šnirel'man's constant. *Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4)*, 22(4):645–706, 1995.
- [Ram09] O. Ramaré. *Arithmetical aspects of the large sieve inequality*, volume 1 of *Harish-Chandra Research Institute Lecture Notes*. Hindustan Book Agency, New Delhi, 2009. With the collaboration of D. S. Ramana.
- [Ram10] O. Ramaré. On Bombieri's asymptotic sieve. *J. Number Theory*, 130(5):1155–1189, 2010.
- [Ram12] O. Ramaré. On long κ -tuples with few prime factors. *Proc. Lond. Math. Soc. (3)*, 104(1):158–196, 2012.
- [Ram13a] O. Ramaré. Explicit estimates for the summatory function of $\Lambda(n)/n$ from the one of $\Lambda(n)$. *Acta Arith.*, 159(2):113–122, 2013.
- [Ram13b] O. Ramaré. From explicit estimates for primes to explicit estimates for the Möbius function. *Acta Arith.*, 157(4):365–379, 2013.
- [Ram14] O. Ramaré. Explicit estimates on the summatory functions of the Möbius function with coprimality restrictions. *Acta Arith.*, 165(1):1–10, 2014.
- [Ram15] O. Ramaré. Explicit estimates on several summatory functions involving the Moebius function. *Math. Comput.*, 84(293):1359–1387, 2015.
- [Ram19] Olivier Ramaré. Corrigendum to explicit estimates on several summatory functions involving the moebius function. *Math. Comput.*, 88(319):2383–2388, 2019.
- [Rem98] R. Remmert. *Classical topics in complex function theory. Transl. by Leslie Kay.*, volume 172. New York, NY: Springer, 1998.
- [Ros41] B. Rosser. Explicit bounds for some functions of prime numbers. *Amer. J. Math.*, 63:211–232, 1941.
- [RR96] O. Ramaré and R. Rumely. Primes in arithmetic progressions. *Math. Comp.*, 65(213):397–425, 1996.
- [RR05] N. Revol and F. Rouillier. Motivations for an Arbitrary Precision Interval Arithmetic and the MPFI Library. *Reliable Computing*, 11(4):275–290, 2005.
- [RS62] J. B. Rosser and L. Schoenfeld. Approximate formulas for some functions of prime numbers. *Illinois J. Math.*, 6:64–94, 1962.

- [RS75] J. B. Rosser and L. Schoenfeld. Sharper bounds for the Chebyshev functions $\theta(x)$ and $\psi(x)$. *Math. Comp.*, 29:243–269, 1975. Collection of articles dedicated to Derrick Henry Lehmer on the occasion of his seventieth birthday.
- [RS03] O. Ramaré and Y. Saouter. Short effective intervals containing primes. *J. Number Theory*, 98(1):10–33, 2003.
- [RT16] E. Roura and A. Travesa. Two independent checkings of the weak Goldbach conjecture up to 10^{27} . *Exp. Math.*, 25(1):79–82, 2016.
- [Rud74] W. Rudin. *Real and complex analysis*. McGraw-Hill Book Co., New York-Düsseldorf-Johannesburg, second edition, 1974. McGraw-Hill Series in Higher Mathematics.
- [Rum93] R. Rumely. Numerical computations concerning the ERH. *Math. Comp.*, 61(203):415–440, S17–S23, 1993.
- [RV] O. Ramaré and G. Kasi Viswanadham. A sharp bilinear form decomposition for primes and Möbius function. Preprint.
- [RV83] H. Riesel and R. C. Vaughan. On sums of primes. *Ark. Mat.*, 21(1):46–74, 1983.
- [Sao98] Y. Saouter. Checking the odd Goldbach conjecture up to 10^{20} . *Math. Comp.*, 67(222):863–866, 1998.
- [Sch18] J. Schur. Einige Bemerkungen zu der vorstehenden Arbeit des Herrn G. Pólya: Über die Verteilung der quadratischen Reste und Nichtreste. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse*, 1918:30–36, 1918.
- [Sch33] L. Schnirelmann. Über additive Eigenschaften von Zahlen. *Math. Ann.*, 107(1):649–690, 1933.
- [Sch69] L. Schoenfeld. An improved estimate for the summatory function of the Möbius function. *Acta Arith.*, 15:221–233, 1969.
- [Sch76] L. Schoenfeld. Sharper bounds for the Chebyshev functions $\theta(x)$ and $\psi(x)$. II. *Math. Comp.*, 30(134):337–360, 1976.
- [SD10] Y. Saouter and P. Demichel. A sharp region where $\pi(x) - \text{li}(x)$ is positive. *Math. Comp.*, 79(272):2395–2405, 2010.
- [Sel42] A. Selberg. On the zeros of Riemann’s zeta-function. *Skr. Norske Vid. Akad. Oslo I.*, 1942(10):59, 1942.
- [Sel72] A. Selberg. Remarks on sieves. Proc. 1972 Number Theory Conf., Univ. Colorado, Boulder 1972, 205–216 (1972), 1972.

- [Sel91] A. Selberg. Lectures on sieves. In *Collected papers, vol. II*, pages 66–247. Springer Berlin, 1991.
- [SG77] G. E. Shilov and B. L. Gurevich. *Integral, measure and derivative: a unified approach. Rev. Engl. ed. Translated from Russian and edited by Richard A. Silverman*. Dover Publications, Mineola, NY, 1977.
- [Sha14] X. Shao. A density version of the Vinogradov three primes theorem. *Duke Math. J.*, 163(3):489–512, 2014.
- [Shi73] G. Shimura. On modular forms of half integral weight. *Ann. Math. (2)*, 97:440–481, 1973.
- [Shu92] F. H. Shu. The Cosmos. In *Encyclopaedia Britannica, Macropaedia*, volume 16, pages 762–795. Encyclopaedia Britannica, Inc., 15 edition, 1992.
- [Sie06] W. Sierpiński. O pewnym zagadnieniu z rachunku funkcji asymptotycznych. *Prace matematyczno-fizyczne*, 1(17):77–118, 1906.
- [Sie32] C. L. Siegel. Über Riemanns Nachlass zur analytischen Zahlentheorie. Quell. Stud. Gesch. Math. B 2, 45–80, 1932.
- [Sie35] C. L. Siegel. Über die Classenzahl quadratischer Zahlkörper. *Acta Arith.*, 1:83–86, 1935.
- [Sim] A. Simonič. Explicit zero density estimate for the Riemann zeta-function near the critical line. Preprint. Available as <https://arxiv.org/abs/1910.08274>.
- [Sin69] R. C. Singleton. Algorithm 357: an efficient prime number generator. *Communications of the ACM*, 12:563–564, 1969.
- [Sou09] K. Soundararajan. Partial sums of the Möbius function. *J. Reine Angew. Math.*, 631:141–152, 2009.
- [Spi70] R. Spira. Residue class characters. *Duke Math. J.*, 37:633–637, 1970.
- [SS90] F. W. Schäfke and A. Sattler. Restgliedabschätzungen für die Stirlingsche Reihe. *Note Mat.*, 10(suppl. 2):453–470, 1990.
- [Ste71] S. B. Stechkin. The zeros of the Riemann zeta-function. *Math. Notes*, 8:706–711, 1971.
- [SW71] E. M. Stein and G. Weiss. Introduction to Fourier analysis on Euclidean spaces. Princeton Mathematical Series. Princeton, N. J.: Princeton University Press. X, 297 p. \$ 15.00 (1971)., 1971.
- [Tao14] T. Tao. Every odd number greater than 1 is the sum of at most five primes. *Math. Comp.*, 83(286):997–1038, 2014.

- [Tar51] A. Tarski. *A decision method for elementary algebra and geometry*. University of California Press, Berkeley and Los Angeles, Calif., 1951. 2nd ed.
- [Tat51] T. Tatuzawa. On a theorem of Siegel. *Jap. J. Math.*, 21:163–178 (1952), 1951.
- [TEH12] T. Tao, E. Croot, III, and H. Helfgott. Deterministic methods to find primes. *Math. Comp.*, 81(278):1233–1246, 2012.
- [Tem10] N. M. Temme. Parabolic cylinder functions. In *NIST Handbook of mathematical functions*, pages 303–319. U.S. Dept. Commerce, Washington, DC, 2010.
- [Tem15] N. M. Temme. *Asymptotic methods for integrals*. Hackensack, NJ: World Scientific, 2015.
- [Ten15] G. Tenenbaum. *Introduction to analytic and probabilistic number theory*. Providence, RI: American Mathematical Society (AMS), 3rd expanded edition, 2015.
- [Tit35] E. C. Titchmarsh. The zeros of the Riemann zeta-function. *Proc. R. Soc. Lond., Ser. A*, 151:234–255, 1935.
- [Tit39] E. C. Titchmarsh. *The theory of functions*. Oxford University Press, Oxford, second edition, 1939.
- [Tit48] E. C. Titchmarsh. *Introduction to the theory of Fourier integrals*. Clarendon Press, Oxford, second edition, 1948.
- [Tit86] E. C. Titchmarsh. The theory of the Riemann zeta-function. 2nd ed., rev. by D. R. Heath-Brown. Oxford Science Publications. Oxford: Clarendon Press. x, 412 pp., 1986.
- [Tru11] T. Trudgian. Improvements to Turing’s method. *Math. Comput.*, 80(276):2259–2279, 2011.
- [Tru12] T. Trudgian. An improved upper bound for the argument of the Riemann zeta-function on the critical line. *Math. Comput.*, 81(278):1053–1061, 2012.
- [Tru14] T. S. Trudgian. An improved upper bound for the argument of the Riemann zeta-function on the critical line. II. *J. Number Theory*, 134:280–292, 2014.
- [Tru15a] T. S. Trudgian. Explicit bounds on the logarithmic derivative and the reciprocal of the Riemann zeta-function. *Funct. Approx. Comment. Math.*, 52(2):253–261, 03 2015.

- [Tru15b] T. S. Trudgian. An improved upper bound for the error in the zero-counting formulae for Dirichlet L -functions and Dedekind zeta-functions. *Math. Comp.*, 84(293):1439–1450, 2015.
- [Tru16] T. Trudgian. Updating the error term in the prime number theorem. *Ramanujan J.*, 39(2):225–234, 2016.
- [Tuc11] W. Tucker. *Validated numerics: A short introduction to rigorous computations*. Princeton University Press, Princeton, NJ, 2011.
- [Tur53] A. M. Turing. Some calculations of the Riemann zeta-function. *Proc. London Math. Soc. (3)*, 3:99–117, 1953.
- [TV03] N. M. Temme and R. Vidunas. Parabolic cylinder functions: examples of error bounds for asymptotic expansions. *Anal. Appl. (Singap.)*, 1(3):265–288, 2003.
- [Vaa85] J. D. Vaaler. Some extremal functions in Fourier analysis. *Bull. Am. Math. Soc., New Ser.*, 12:183–216, 1985.
- [van37] J. G. van der Corput. Sur l’hypothèse de Goldbach pour presque tous les nombres pairs. *Acta Arith.*, 2:266–290, 1937.
- [Var06] V. S. Varadarajan. *Euler through time: a new look at old themes*. Providence, RI: American Mathematical Society, 2006.
- [Vau77a] R. C. Vaughan. On the estimation of Schnirelman’s constant. *J. Reine Angew. Math.*, 290:93–108, 1977.
- [Vau77b] R.-C. Vaughan. Sommes trigonométriques sur les nombres premiers. *C. R. Acad. Sci. Paris Sér. A-B*, 285(16):A981–A983, 1977.
- [Vau80] R. C. Vaughan. Recent work in additive prime number theory. In *Proceedings of the International Congress of Mathematicians (Helsinki, 1978)*, pages 389–394. Acad. Sci. Fennica, Helsinki, 1980.
- [Vau97] R. C. Vaughan. *The Hardy-Littlewood method*, volume 125 of *Cambridge Tracts in Mathematics*. Cambridge University Press, Cambridge, second edition, 1997.
- [vdH09] J. van der Hoeven. Ball arithmetic. <https://hal.archives-ouvertes.fr/hal-00432152>, November 2009.
- [Vin37] I. M. Vinogradov. A new method in analytic number theory (Russian). *Tr. Mat. Inst. Steklova*, 10:5–122, 1937.
- [Vin47] I.M. Vinogradov. The method of trigonometrical sums in the theory of numbers (Russian). *Tr. Mat. Inst. Steklova*, 23:3–109, 1947.
- [Vin54] I. M. Vinogradov. *The method of trigonometrical sums in the theory*

- of numbers.* Interscience Publishers, London and New York, 1954.
Translated, revised and annotated by K. F. Roth and Anne Davenport.
- [Vin58] I. M. Vinogradov. A new estimate of the function $\zeta(1 + it)$. *Izv. Akad. Nauk SSSR. Ser. Mat.*, 22:161–164, 1958.
 - [vLR65] J. H. van Lint and H.-E. Richert. On primes in arithmetic progressions. *Acta Arith.*, 11:209–216, 1965.
 - [vM05] H. von Mangoldt. Zur Verteilung der Nullstellen der Riemannschen Funktion $\xi(t)$. *Math. Ann.*, 60:1–19, 1905.
 - [Vor03] G. Voronoï. Sur un problème du calcul des fonctions asymptotiques. *J. Reine Angew. Math.*, 126:241–282, 1903.
 - [Wal] K. Walisch. primesieve: fast C/C++ prime number generator. <http://primesieve.org>. Accessed: 2016-05-16.
 - [Web69] H. Weber. Ueber die Integration der partiellen Differentialgleichung: $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + k^2u = 0$. *Math. Ann.*, 1:1–36, 1869.
 - [Wed03] S. Wedeniwski. ZetaGrid - Computational verification of the Riemann hypothesis. Conference in Number Theory in honour of Professor H. C. Williams, Banff, Alberta, Canada, May 2003.
 - [Wei52] A. Weil. Sur les “formules explicites” de la théorie des nombres premiers. *Meddel. Lunds Univ. Mat. Sem.*, Suppl.-band M. Riesz, 252–265, 1952.
 - [Wei84] A. Weil. *Number theory: An approach through history. From Hammurapi to Legendre*. Birkhäuser Boston, Inc., Boston, MA, 1984.
 - [Wei09] A. Weil. *Œuvres scientifiques. Collected papers. Vol. II (1951–1964). Paperback reprint of the 1979 edition*. Berlin: Springer, paperback reprint of the 1979 edition edition, 2009.
 - [Wes22] A. E. Western. Note on the number of primes of the form $n^2 + 1$. *Proc. Camb. Philos. Soc.*, 21:108–109, 1922.
 - [Whi03] E. T. Whittaker. On the functions associated with the parabolic cylinder in harmonic analysis. *Proc. London Math. Soc.*, 35:417–427, 1903.
 - [Wig20] S. Wigert. Sur la théorie de la fonction $\zeta(s)$ de Riemann. *Ark. Mat.*, 14:1–17, 1920.
 - [Won01] R. Wong. *Asymptotic approximations of integrals*, volume 34 of *Classics in Applied Mathematics*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2001. Corrected reprint of the 1989 original.

- [Wym64] M. Wyman. The method of Laplace. *Trans. Royal Soc. Canada*, 2:227–256, 1964.
- [Zha14] Y. Zhang. Bounded gaps between primes. *Ann. of Math.* (2), 179(3):1121–1174, 2014.
- [Zin97] D. Zinoviev. On Vinogradov’s constant in Goldbach’s ternary problem. *J. Number Theory*, 65(2):334–358, 1997.