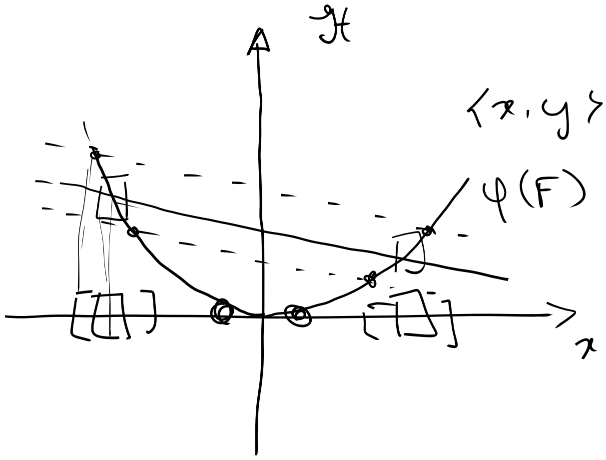


$\langle x, y \rangle = xy$

$x \longmapsto (x, \beta x^2)$

$\mathbb{R} = F \longrightarrow \varphi(F)$

$x \in \mathbb{R}^d \quad K(x,y) = \langle x,y\rangle + \beta \langle x,y\rangle^2$

$$= \sum x_i y_i + \beta \sum_{i,j} x_i y_i x_j y_j$$

$$\varphi : \underset{=(x_i)_{i\in I}}{x} \longmapsto (x_i, x_i^2, x_i x_j, i \neq j).$$

K noyau positif : $x^1, \dots, x^m \in F^m$
la matrice $K(x^i, x^j)$ est symétrique et positive.

$$\varphi(x) = \{ y \longmapsto K(x,y) : F \to \mathbb{R} \} = \{K_x\}$$

$$\mathcal{H}_0 = \operatorname{vect}\{K_x\}$$

$$\langle \underbrace{\sum \alpha_i K_{x_i}}_{f}, \underbrace{\sum \beta_j K_{x_j}}_{g} \rangle = \sum \alpha_i \beta_j K(x_i, x_j)$$

$$= \sum \beta_j f(x_j)$$

$$= \sum \alpha_i g(x_i)$$

$\mathbb{R}^I$

$$K_\lambda(x,y) = (e^\lambda - 1)^{-1} \left[\exp(\lambda \langle x,y\rangle) - 1\right] \quad \lambda > 0$$

$$= (e^\lambda - 1)^{-1} \sum_{k=1}^{+\infty} \frac{\lambda^k \langle x,y\rangle^k}{k!}$$

$$\xrightarrow[\lambda \to 0]{} \langle x,y\rangle$$

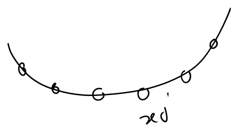
$\underset{ev}{F} \longrightarrow \varphi(F)$ variété courbe dans $\mathcal{H}$

$\{f(i), i \in I\} \longrightarrow x_i = \left(f_i / \sum_j f_j \right)^{1/2}$

image

$x^j \in S_F$

$j = 1, \dots, n$



$V_1 = \text{vect}\{x^j, j = 1, \dots, n\}$

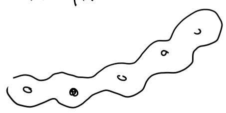
$V_1 \cap S_F$

$\varphi_\lambda : F \longrightarrow \mathcal{H}$ à l'aide K_λ

$V_2 = \text{vect}\{\varphi_\lambda(x^j), j = 1, \dots, n\}$ ds $\mathcal{H}$

$(V_2 \cap S_{\mathcal{H}}) \cap \varphi_\lambda(F) = \{\varphi_\lambda(x^j), j = 1, \dots, n\}$

$N(V_2 \cap S_{\mathcal{H}}, \varepsilon) \cap \varphi_\lambda(F)$



$\tilde{x}^j = x^j + g_j \quad g_j \in V_1$

$x_{i,j} = u s v'$ u v orthogonales, s diagonale

$= \sum_k u_{i,k} (sv')_{k,j}$

$\check{x}^j = \|\tilde{x}^j\|^{-1} \tilde{x}^j$

donne idée de la "taille" de $V_1 \cap S_F$

$x x' = u s^2 u'$

$x' x = v s^2 v'$

$\check{x}^j \longmapsto \underset{\mathcal{H}}{\varphi(\check{x}^j)} \longmapsto \pi_{S_{V_2}}(\check{x}^j)$

$\downarrow$

$\bar{x}^j = \pi_F(\pi_{S_{V_2}}(\check{x}^j))$

$x \in S_F, \; f \in S_{\mathcal{H}}$

$\langle f, K_x \rangle_{\mathcal{H}} = f(x)$

$\|f - K_x\|_{\mathcal{H}} = 2(1 - \langle f, K_x \rangle)$

$\pi_{S_F}(f) = \arg\max_x f(x) = \arg\max_x \sum_j \alpha_j \exp[\lambda \langle y_j, x \rangle]$

$f(x) = \sum_j \alpha_j K_\lambda(y_j, x) = \sum_j \alpha_j \exp \lambda \langle y_j, x \rangle + \text{cte}$

$\sum_j \alpha_j \exp(\lambda \langle y_j, x \rangle) y_j = \beta x$

calcul itératif de $\pi_{S_F}^{\mathcal{H}}(f)$