ERRATUM

The following proposition corrects the part of the proof of Proposition 5.7 of the article
O. Debarre, R. Fahlaoui, Abelian Varieties In $W_d^r(C)$ And Points Of Bounded Degrees On Algebraic Curves, *Comp. Math.* **88** (1993), 235–249,
which is incomplete (to wit the case $d = 4$).

Proposition.— *Assume $d = 4$ and C general in $|L|$. Then C has no g^1_4.*

Proof. Let A be a base-point-free g^1_δ on C, with $\delta \le 4$. Following [DF] (5.12), one constructs a rank 2 vector bundle T on S that fits into an exact sequence

$$(*) \qquad 0 \to H^0(A)^* \otimes \mathcal{O}_S \to T \to \mathcal{O}_C(C - A) \to 0 \ .$$

Note that $H^2(T) = 0$ and $\chi(T) = 10 - \delta$ by Riemann–Roch. If $h = h^0(T) > 6$, the kernel of the map $\wedge^2 H^0(T) \to H^0(\wedge^2 T) \simeq H^0(S, C) \simeq \mathbf{C}^{10}$ meets the $(2h-3)$-dimensional set of decomposable vectors off the origin. One proceeds as in [DF] (where the numbers at the top of page 246 are all wrong) to show that there exists a divisor D on S that fits into an exact sequence

$$0 \to \mathcal{O}_S(D) \to T \to \mathcal{I}_Z(C - D) \to 0 \ ,$$

where Z is a finite subscheme of S; moreover, either $D \sim 2H$ or $D \sim 3H - F$. Then, $h^0(T) \le h^0(D) + h^0(C - D) = 5$, which is a contradiction (this remark avoids the lengthy proof in [DF]).

It follows that $h^0(T) = 6$, $h^1(T) = 0$ and $\delta = 4$. Assume first that there is a non-zero morphism $u : T \to T \otimes \omega_S$. We argue as in [L]: since $H^0(\omega_S^2) = 0$, the morphism $\wedge^2 u$ vanishes hence u drops rank everywhere. Then $N = (\operatorname{Im} u)^{**}$ is a line bundle on S which is a subsheaf of $T \otimes \omega_S$; there is a morphism $T \to N$ which is surjective off a finite subset of S. Note that by Riemann–Roch, one has $h^0(\mathcal{O}_C(C - A)) \ge 5 > 2 = h^1(H^0(A)^* \otimes \mathcal{O}_S)$, hence the exact sequence $(*)$ shows that T is generated by global sections off a finite subset of S, hence so is N. It follows that either $h^0(S, N) \ge 2$, or $N \simeq \mathcal{O}_S$; but the latter cannot occur since $\operatorname{Hom}(T, \mathcal{O}_S) = 0$. Tensoring $(*)$ by $\omega_S \otimes N^*$, we see that $H^0(T \otimes \omega_S \otimes N^*) \ne 0$ implies $H^0(\omega_C \otimes N^* \otimes \mathcal{O}_C(-A)) \ne 0$ and in particular $(5H - F) \cdot (3H - N) \ge 4$. Furthermore, there is an exact sequence

$$0 \to N \to T \otimes \omega_S \to \mathcal{I}_Z(\omega_S^{\otimes 2} \otimes N^*(C)) \to 0 \ ,$$

which implies $N \cdot (H + F - N) \le c_2(T \otimes \omega_S) = 0$. A case-by-case analysis shows that the only possibility is $N \sim 2H$; but then $H^0(\omega_C \otimes N^*(-A)) \ne 0$ implies $A \equiv \overline{H}_x$, which is not a pencil.

Hence $\operatorname{Hom}(T, T \otimes \omega_S)$ vanishes, and so does $H^2(\operatorname{End} T)$ by duality. Dualizing $(*)$ yields

$$0 \to T^* \to H^0(A) \otimes \mathcal{O}_S \to \mathcal{O}_C(A) \to 0 \ .$$

Tensoring by T, we get $H^1(T \otimes A) = 0$. We now follow another construction of [L], where a moduli space P is constructed which parametrizes triples (C, A, l), where C is a smooth curve in $|L|$, A is a base-point-free g^1_4 on C, and l is a surjective morphism $H \otimes_{\mathbf{C}} \mathcal{O}_S \to A$ which induces an isomorphism on global sections, two such morphisms being identified if they differ by multiplication by a non-zero scalar. Let $\pi : P \to |L|$ be the forgetful morphism. The tangent space to P at (C, A, l) is identified with the kernel $\tilde{H}^0(T \otimes A)$ of the map $H^0(T \otimes A) \to H^1(\operatorname{End} T) \xrightarrow{\operatorname{Tr}} H^1(\mathcal{O}_S)$; the tangent space to $|L|$ at C is identified with the kernel $\tilde{H}^0(C, L)$ of the map $H^0(C, L) \to H^1(\mathcal{O}_S)$. There is an exact sequence ([L], page 304)

$$\tilde{H}^0(T \otimes A) \xrightarrow{T_{(C,A,l)}\pi} \tilde{H}^0(C, L) \longrightarrow (\operatorname{Ker} \mu)^* \longrightarrow \tilde{H}^1(T \otimes A)$$

where $\mu : H^0(A) \otimes H^0(\omega_C \otimes A^*) \to H^0(\omega_C)$ is the Petri map. By the base-point-free pencil trick, its kernel is isomorphic to $H^0(\omega_C \otimes (A^{\otimes 2})^*)$, which has by Riemann–Roch dimension at least $h^0(A^{\otimes 2}) - 2 > 0$. Since $H^1(T \otimes A)$ vanishes, $T_{(C,A,l)}\pi$ is not surjective, hence neither is π by generic smoothness. This shows that there is no g^1_4 on a generic C in $|L|$, and finishes the proof of the proposition. ∎

REFERENCES

[DF] Debarre, O., Fahlaoui, R., Abelian varieties In $W^r_d(C)$ and points of bounded degrees on algebraic curves, *Comp. Math.* **88** (1993), 235–249,

[L] Lazarsfeld, R., Brill–Noether–Petri without Degenerations, *J. Diff. Geom.* **23** (1986), 299–307.