Erratum for "Fulton-Hansen and Barth-Lefschetz Theorems for Subvarieties of Abelian Varieties"

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As F. Laytimi pointed out, it is not true in general, as claimed at the top of page 195 of [D], that the tensor product of a k-ample vector bundle and a nef line bundle is again k-ample: on a curve, the structure sheaf is 1-ample, but its tensor product with a non-torsion numerically trivial line bundle is not 1-ample, since no non-trivial tensor power has non-zero sections. In other words, even for line bundles, being k-ample is not a numerical property for $k > 0$ (see also [So], (1.4.2)).

To fix the argument of [D], one needs to go back to the proof of [So], Proposition (1.12), which says that if L is a k-ample line bundle on a projective d-dimensional manifold X, we have

$$H^q(X, \Omega_X^q \otimes L) = 0 \qquad \text{for } p + q > d + k$$

(there is a sign error in [So]). The same proof, which ultimatey relies on the Kodaira-Nakano vanishing theorem (of which it is a generalization), indeed yields the following.

Proposition. *Let X be a projective d-dimensional manifold, let L be a k-ample line bundle on X, and let N be a nef line bundle on X. Then,*

$$H^q(X, \Omega_X^q \otimes L \otimes N) = 0 \qquad \textit{for } p + q > d + k.$$

We can then deduce by Schneider's argument ([S]; [L], Theorem 7.3.5 and Remark 7.3.6, and especially the top of page 94) the following extension of Le Potier's theorem ([LP]; [So], Proposition (1.13)).

Proposition. *Let X be a projective d-dimensional manifold, let E be a k-ample vector bundle of rank r on X, and let N be a nef line bundle on X. Then,*

$$H^q(X, \Omega_X^q \otimes E^\vee \otimes N^\vee) = 0 \qquad \textit{for } p + q \leq d - k - r.$$

Using this statement instead of the weaker Theorem 4.2 of [D], the proofs of Theorem 4.4 and Theorem 4.5 of [D] apply *verbatim.*

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References

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