

# Erratum for “Fulton-Hansen and Barth-Lefschetz Theorems for Subvarieties of Abelian Varieties”

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As F. Laytimi pointed out, it is not true in general, as claimed at the top of page 195 of [D], that the tensor product of a  $k$ -ample vector bundle and a nef line bundle is again  $k$ -ample: on a curve, the structure sheaf is 1-ample, but its tensor product with a non-torsion numerically trivial line bundle is not 1-ample, since no non-trivial tensor power has non-zero sections. In other words, even for line bundles, being  $k$ -ample is not a numerical property for  $k > 0$  (see also [So], (1.4.2)).

To fix the argument of [D], one needs to go back to the proof of [So], Proposition (1.12), which says that if  $L$  is a  $k$ -ample line bundle on a projective  $d$ -dimensional manifold  $X$ , we have

$$H^q(X, \Omega_X^q \otimes L) = 0 \quad \text{for } p + q > d + k$$

(there is a sign error in [So]). The same proof, which ultimately relies on the Kodaira-Nakano vanishing theorem (of which it is a generalization), indeed yields the following.

**Proposition.** *Let  $X$  be a projective  $d$ -dimensional manifold, let  $L$  be a  $k$ -ample line bundle on  $X$ , and let  $N$  be a nef line bundle on  $X$ . Then,*

$$H^q(X, \Omega_X^q \otimes L \otimes N) = 0 \quad \text{for } p + q > d + k.$$

We can then deduce by Schneider’s argument ([S]; [L], Theorem 7.3.5 and Remark 7.3.6, and especially the top of page 94) the following extension of Le Potier’s theorem ([LP]; [So], Proposition (1.13)).

**Proposition.** *Let  $X$  be a projective  $d$ -dimensional manifold, let  $E$  be a  $k$ -ample vector bundle of rank  $r$  on  $X$ , and let  $N$  be a nef line bundle on  $X$ . Then,*

$$H^q(X, \Omega_X^q \otimes E^\vee \otimes N^\vee) = 0 \quad \text{for } p + q \leq d - k - r.$$

Using this statement instead of the weaker Theorem 4.2 of [D], the proofs of Theorem 4.4 and Theorem 4.5 of [D] apply *verbatim*.

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## References

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