Erratum for "Varieties with ample cotangent bundle"

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O. Benoist pointed out several errors and inaccuracies in the article [D].

1) Proposition 5 is wrong as stated and should read as follows.

Proposition 5 *The normal bundle of a smooth nondegenerate subvariety of an abelian variety is nef and big.*

The converse is obviously wrong (consider any smooth curve C in a simple abelian variety A of dimension at least 2; if B is any nonzero abelian variety, the normal bundle of C in $A \times B$ is nef and big, although C is degenerate). The error occurs in the last paragraph of the proof: the tangent spaces to X along a general fiber of $\pi|_X$ are indeed contained in a fixed hyperplane, but this hyperplane is not general, so we cannot conclude that the fiber is finite.

As a result, the second implication of Proposition 6 is unproved. That proposition should read as follows.

Proposition 6 *Let X be a smooth subvariety of an abelian variety A. If Ω_X is ample, $N_{X/A}$ is nef and big.*

2) Lemma 12 is wrong as stated: if Y is an integral surface with isolated singularities in $\mathbf{P}^n$ which is not Cohen-Macaulay at some point p, the local ring $\mathscr{O}_{Y,p}$ has depth 1. If F is any hypersurface in $\mathbf{P}^n$ which passes through p but does not contain Y, the local ring $\mathscr{O}_{Y\cap F,p}$ has depth 0, hence is not reduced: the intersection $Y \cap F$ has an embedded point at p. Therefore, the variety $\mathscr{V}_e(Y)$ has codimension 1 in $\mathscr{V}_{e,n}$ for any $e \geq 1$.

O. Benoist proved in [B], Théorème 1.5, that this is essentially the only case when things go wrong, and that Lemma 12 holds when Y is for example normal. To correct the proofs of Theorems 7, 8, and 9 of [D], which all rely on this lemma, we need a slightly modified version of Benoist's theorem which follows easily from Théorème 1.4 of [B].

Lemma 12 (O. Benoist) *Let Y be an integral subscheme of $\mathbf{P}^n$ of dimension at least 2, let $\nu : \widehat{Y} \to Y$ be its normalization, and let $\mathscr{V}_{e,n}$ be the projective space of hypersurfaces of degree e in $\mathbf{P}^n$. The codimension of the complement $\mathscr{V}_e(Y)$ of*

$$\{F \in \mathscr{V}_{e,n} \mid \nu^*F \text{ is integral of codimension } 1 \text{ in } \widehat{Y}\ \}$$

in $\mathscr{V}_{e,n}$ is at least $e-1$.

The proofs of Theorems 7, 8, and 9 of [D] can then be easily adapted. For example, in the proof of Theorem 7, just replace diagram (4) with:

$$\begin{array}{ccc} H^0(H, L_c^e|_H) & \longrightarrow & H^1(A, \mathscr{O}_A) \\ \downarrow & & \downarrow{\scriptstyle \rho} \\ H^0(\nu^*H, \nu^*L_c^e) & \longrightarrow & H^1(\widehat{Y}, \mathscr{O}_{\widehat{Y}}). \end{array}$$

References

[B] Benoist, O., Le théorème de Bertini en famille, *Bull. Soc. math. Fr.* **139** (2011), 555–569.

[D] Debarre, O., Varieties with ample cotangent bundle, *Compos. Math.* **141** (2005), 1445–1459.