

**Errata for**  
**“Higher-Dimensional Algebraic Geometry”**  
**by Olivier Debarre**

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**page 2**

- line 31, for  
“A variety is an integral scheme”  
read  
“A variety is a geometrically integral scheme”

**page 4**

- line 20, for  
“connected reduced”  
read  
“geometrically connected and reduced”

**page 9**

- line 12, for  
“let  $C$  be a curve.”  
read  
“let  $C$  be an irreducible curve.”

**page 11**

- line 5, for  
“the set of classes of effective 1-cycles.”  
read  
“generated by classes of effective 1-cycles.”

If  $Y$  is another proper scheme, any morphism  $\pi : X \rightarrow Y$  induces linear maps

$$\pi^* : N^1(Y)_{\mathbf{Z}} \rightarrow N^1(X)_{\mathbf{Z}} \quad \text{and} \quad \pi_* : N^1(X)_{\mathbf{Z}} \rightarrow N^1(Y)_{\mathbf{Z}}$$

defined by (see § 1.9)

$$\pi^*([D]) = [\pi^*(D)] \quad \text{and} \quad \pi_*([C]) = [\pi_*(C)] = \deg(C \xrightarrow{\pi} \pi(C)) [\pi(C)]$$

They satisfy the projection formula (see (1.5))

$$\pi^*(d) \cdot c = d \cdot \pi_*(c)$$

- line -13, for

“ $\text{Spec}(\pi_*\mathcal{O}_Y)$ ”

read

“ $\text{Spec}(\pi_*\mathcal{O}_X)$ ”

**page 13**

- line 6, for

“ $X^0 = f^{-1}(Y^0)$ ”

read

“ $X^0 = \pi^{-1}(Y^0)$ ”

**page 22**

- line -10, for

“the left-hand side of this equality is nonnegative”

read

“the left-hand side of this equality is nonpositive”

**page 27**

- line -11, for

“nine points”

read

“nine general points”

**page 31**

- line 10, as of

“Recall also...”

replace the end of the proof of Proposition 1.43 by

“ Each component of  $E$  has codimension 1 (1.40) hence, by shrinking  $Y$ , we may assume that  $Y$  and  $E$  are smooth and irreducible. Set  $U_0 = X - \text{Sing}(\overline{\pi(E)})$ , so that the closure in  $U_0$  of the image of  $E \cap \pi^{-1}(U_0)$  is smooth, of codimension at least 2. Let  $\varepsilon_1 : X_1 \rightarrow U_0$  be its blow-up; by the universal property of blow-ups ([H1], II, prop. 7.14), there exists a factorization

$$\pi|_{V_1} : V_1 \xrightarrow{\pi_1} X_1 \xrightarrow{\varepsilon_1} U_0 \subset X$$

where the complement of  $V_1 = \pi^{-1}(U_0)$  in  $Y$  has codimension at least 2 and  $\overline{\pi_1(E \cap V_1)}$  is contained in the support of the exceptional divisor of  $\varepsilon_1$ . If the codimension of  $\overline{\pi_1(E \cap V_1)}$  in  $X_1$  is at least 2, the divisor  $E \cap V_1$  is contained in the exceptional locus of  $\pi_1$  and, upon replacing  $V_1$  by the complement  $V_2$  of a closed subset of codimension at least 2 and  $X_1$  by an open subset  $U_1$ , we may repeat the construction. After  $i$  steps, we get a factorization

$$\pi : V_i \xrightarrow{\pi_i} X_i \xrightarrow{\varepsilon_i} U_{i-1} \subset X_{i-1} \xrightarrow{\varepsilon_{i-1}} \cdots \xrightarrow{\varepsilon_2} U_1 \subset X_1 \xrightarrow{\varepsilon_1} U_0 \subset X$$

as long as the codimension of  $\overline{\pi_{i-1}(E \cap V_{i-1})}$  in  $X_{i-1}$  is at least 2, where  $V_i$  is the complement in  $Y$  of a closed subset of codimension at least 2. Let  $E_j \subset X_j$

be the exceptional divisor of  $\varepsilon_j$ . We have

$$\begin{aligned} K_{X_i} &= \varepsilon_i^* K_{U_{i-1}} + c_i E_i \\ &= (\varepsilon_1 \circ \cdots \circ \varepsilon_i)^* K_X + c_i E_i + c_{i-1} E_{i,i-1} + \cdots + c_1 E_{i,1} \end{aligned}$$

where  $E_{i,j}$  is the inverse image of  $E_j$  in  $X_i$  and

$$c_i = \text{codim}_{X_{i-1}}(\overline{\pi_{i-1}(E \cap V_{i-1})}) - 1 > 0$$

([H1], II, Ex. 8.5). Since  $\pi_i$  is birational,  $\pi_i^* \mathcal{O}_{X_i}(K_{X_i})$  is a subsheaf of  $\mathcal{O}_{V_i}(K_{V_i})$ . Moreover, since  $\pi_j(E \cap V_j)$  is contained in the support of  $E_j$ , the divisor  $\pi_j^* E_j - E|_{V_j}$  is effective, hence so is  $E_{i,j} - E|_{V_i}$ .

It follows that  $\mathcal{O}_Y(\pi^* K_X + (c_i + \cdots + c_1)E)|_{V_i}$  is a subsheaf of  $\mathcal{O}_{V_i}(K_{V_i}) = \mathcal{O}_Y(K_Y)|_{V_i}$ . Since  $Y$  is normal and the complement of  $V_i$  in  $Y$  has codimension at least 2,  $\mathcal{O}_Y(\pi^* K_X + (c_i + \cdots + c_1)E)$  is also a subsheaf of  $\mathcal{O}_Y(K_Y)$ . Since there are no infinite ascending sequences of subsheaves of a coherent sheaf on a Noetherian scheme, the process must terminate at some point:  $\overline{\pi_i(E \cap V_i)}$  is a divisor in  $X_i$  for some  $i$ , hence  $E \cap V_i$  is not contained in the exceptional locus of  $\pi_i$  (by 1.40 again). The morphism  $\pi_i$  then induces a birational isomorphism between  $E \cap V_i$  and  $E_i$ , and the latter is ruled: more precisely, through every point of  $E_i$  there is a rational curve contracted by  $\varepsilon_i$ . This proves the proposition.”

**page 33**

- line 14, for

“The rational map  $\pi^{-1} : S \dashrightarrow Y$  might not be a morphism, but its indeterminacies can be resolved by blowing-up a finite number of points”

read

“The indeterminacies of the rational map  $\pi^{-1} : S \dashrightarrow Y$  can be resolved by blowing-up a finite number of points”

**page 34**

- line 15, for

“one may only say by Proposition 1.45 that there is no birational morphism from  $X_1$  to another member of  $\mathcal{C}$ .”

read

“one may only say by Proposition 1.45 that any birational morphism from  $X_1$  to another member of  $\mathcal{C}$  is an isomorphism.”

**page 37**

- line 6, for

“a given smooth projective curve  $C$  to a given smooth quasi-projective variety.”

read

“a fixed projective curve  $C$  to a fixed smooth quasi-projective variety.”

**page 38**

- line 8, replace  $n$  by  $N$ .

- line 15, in equation (2.1), replace  $u$  by  $U$  and  $v$  by  $V$ .

**page 39**

- line 9, for

“Let now  $X$  be a subscheme”

read

“Let now  $X$  be a (closed) subscheme”

- line 15, for

“of quasi-projective schemes.”

read

“of quasi-projective schemes. The same conclusion holds for any quasi-projective variety  $X$ .”

**page 40**

- line 2, for

“and a polynomial  $P$ ”

read

“and a polynomial  $P$  with rational coefficients”

- between lines 9 and 10, add a new paragraph:

“The fact that  $Y$  is projective is essential in this construction: the space  $\text{Mor}(\mathbf{A}^1, \mathbf{A}^N)$  is *not* a disjoint union of quasi-projective varieties.”

- line 12, for

$$f^{\text{univ}} : Y \times \text{Mor}(Y, X) \rightarrow X \times \text{Mor}(Y, X)$$

such that, for each point  $t$  of  $\text{Mor}(Y, X)$ , the morphism  $f_t^{\text{univ}} : Y \rightarrow X$  is the morphism”

read

$$f^{\text{univ}} : Y \times \text{Mor}(Y, X) \rightarrow X$$

such that, for each point  $t$  of  $\text{Mor}(Y, X)$ , the morphism  $f_t^{\text{univ}} : Y \times \{t\} \rightarrow X$  is the morphism”

- line 22, for

“

- $T$ -morphisms  $f : Y \times T \rightarrow X \times T$

”

obtained by sending  $\varphi$  to the pull-back

$$f(y, t) = (pr_1 \circ f^{\text{univ}}(y, \varphi(t)), t)$$

of  $f^{\text{univ}}$ . Given  $f$  (or  $\varphi$ ), we will call the morphism

$$\begin{array}{ccc} \text{ev} : Y \times T & \longrightarrow & X \\ (y, t) & \longmapsto & f_t(y) = \varphi(t)(y) \end{array}$$

the *evaluation map*.”

read

“ • morphisms  $f : Y \times T \rightarrow X$

obtained by sending  $\varphi$  to the pull-back

$$f(y, t) = f^{\text{univ}}(y, \varphi(t))$$

of  $f^{\text{univ}}$ . Given  $\varphi$ , we will call the morphism  $f$  the *evaluation map* and often denote it by  $\text{ev}$ .”

• line -5, for

“the universal morphism being

$$\begin{array}{ccc} f^{\text{univ}} : \{\star\} \times X & \longrightarrow & X \times X \\ (\star, x) & \longmapsto & (x, x) \end{array}$$

Similarly, when  $Y$  is a reduced scheme of dimension 0, the scheme  $\text{Mor}(Y, X)$  is isomorphic to  $X^Y$ . ”

read

“the universal morphism being

$$\begin{array}{ccc} f^{\text{univ}} : \{\star\} \times X & \longrightarrow & X \\ (\star, x) & \longmapsto & x \end{array}$$

Similarly, when  $Y$  is a reduced scheme of finite length, the scheme  $\text{Mor}(Y, X)$  is isomorphic to  $X^Y$ . ”

#### page 41

• line 9, for

“which parametrizes rational curves on  $X$  whose image is a line”

read

“that parametrizes morphisms  $\mathbf{P}^1 \rightarrow X$  whose image is a line”

• line 13, for

“ $T = \text{Spec } \mathbf{k}[\varepsilon]/(\varepsilon^2)$  to  $\text{Mor}(Y, X)$  with image  $[f]$  ([H1], II, Ex. 2.8), hence extensions of  $f$  to  $T$ -morphisms

$$f_\varepsilon : Y \times \text{Spec } \mathbf{k}[\varepsilon]/(\varepsilon^2) \rightarrow X \times \text{Spec } \mathbf{k}[\varepsilon]/(\varepsilon^2)$$

read

“ $\text{Spec } \mathbf{k}[\varepsilon]/(\varepsilon^2)$  to  $\text{Mor}(Y, X)$  with image  $[f]$  ([H1], II, Ex. 2.8), hence extensions of  $f$  to morphisms

$$f_\varepsilon : Y \times \text{Spec } \mathbf{k}[\varepsilon]/(\varepsilon^2) \rightarrow X$$

#### page 42

• line 14, for

“**Theorem 2.6** *Let  $X$  and  $Y$  be projective varieties and let  $f : Y \rightarrow X$  be a morphism such that  $X$  is smooth along  $f(Y)$ .*”

read

“**Theorem 2.6** Let  $X$  be a quasi-projective variety, let  $Y$  be a projective variety, and let  $f : Y \rightarrow X$  be a morphism such that  $X$  is smooth along  $f(Y)$ .”

- line -5, for

“ The canonical morphism  $\text{Spec}(R/I) \rightarrow \text{Mor}(Y, X)$  corresponds to an extension  $f_{R/I} : Y \times \text{Spec}(R/I) \rightarrow X \times \text{Spec}(R/I)$  of  $f$ . Since  $I^2 \subset \mathfrak{m}I$ , the obstruction to extending it to a morphism  $f_{R/\mathfrak{m}I} : Y \times \text{Spec}(R/\mathfrak{m}I) \rightarrow X \times \text{Spec}(R/\mathfrak{m}I)$  lies by Lemma 2.7 below in”

read

“ The canonical morphism  $\text{Spec}(R/I) \rightarrow \text{Mor}(Y, X)$  corresponds to an extension  $f_{R/I} : Y \times \text{Spec}(R/I) \rightarrow X$  of  $f$ . Since  $I^2 \subset \mathfrak{m}I$ , the obstruction to extending it to a morphism  $f_{R/\mathfrak{m}I} : Y \times \text{Spec}(R/\mathfrak{m}I) \rightarrow X$  lies by Lemma 2.7 below in”

### page 43

- line 13, for

“Let  $R$  be a finitely generated local  $\mathbf{k}$ -algebra”

read

“Let  $R$  be a Noetherian local  $\mathbf{k}$ -algebra”

- line 15, for

$$f_{R/I} : Y \times \text{Spec}(R/I) \rightarrow X \times \text{Spec}(R/I)$$

be an extension of  $f$ . Assume  $X$  is smooth along the image of  $f$ . The obstruction to extending  $f_{R/I}$  to a morphism

$$f_R : Y \times \text{Spec}(R) \rightarrow X \times \text{Spec}(R)$$

read

$$f_{R/I} : Y \times \text{Spec}(R/I) \rightarrow X$$

be an extension of  $f$ . Assume  $X$  is smooth along the image of  $f$ . The obstruction to extending  $f_{R/I}$  to a morphism

$$f_R : Y \times \text{Spec}(R) \rightarrow X$$

### page 44

- line 2, for

“([H1], II, prop. 8.2A)”

read

“([Bo], X, § 7, n° 10, déf. 2)”

- line 16, for

“is necessary and sufficient for a global extension to exist”

read

“is necessary and sufficient for a global extension to exist (on a separated Noetherian scheme, the cohomology of a coherent sheaf is isomorphic to its Čech cohomology relative to any open affine covering; [H1], III, Theorem 4.5).”

• line 22, for

“Let  $\sigma$  be a section of  $\pi$ . If  $a$  and  $b$  are in  $\mathfrak{m}$ , we have  $\sigma \circ \pi(a) = a + a'$  and  $\sigma \circ \pi(b) = b + b'$ , where  $a'$  and  $b'$  are in  $I$ . Then”

read

“Let  $\sigma$  be a section of  $\pi$ : if  $a$  and  $b$  are in  $R$ , we can write  $\sigma \circ \pi(a) = a + a'$  and  $\sigma \circ \pi(b) = b + b'$ , where  $a'$  and  $b'$  are in  $I$ . If  $a$  and  $b$  are in  $\mathfrak{m}$ , we have”

#### page 45

• line 3, for

“We will need a slightly more general situation:”

read

“Again,  $X$  and  $Y$  are varieties over a field  $\mathbf{k}$ , with  $Y$  projective and  $X$  quasi-projective. We will need to study a slightly more general situation:”

• line 12, for

“Note that when  $B$  is finite,”

read

“Note that when  $B$  is finite and  $X$  is smooth,”

• line 16, for

“locally at a point  $[f]$  such that  $X$  is smooth along  $f(Y)$ , *the scheme  $\text{Mor}(Y, X; g)$  can be defined*”

read

“around a point  $[f]$  such that  $X$  is smooth along  $f(Y)$ , *the scheme  $\text{Mor}(Y, X; g)$  can be locally defined*”

• line -7, for

“ $Y$  is a curve  $C$  (and  $B$  is finite):”

read

“ $Y$  is a curve  $C$  (as usual, reduced connected proper over a field) and  $B$  is finite:”

#### page 46

• line 3, for

“over an irreducible base  $S$ ”

read

“over an irreducible scheme  $S$  of finite type over a field”

• line 4, for

“and a morphism”

read

“, a point  $s$  of  $S$ , and a  $k(s)$ -morphism”

- line -4 (footnote), for the references

“(1.7.1), (1.7.2), (1.7.3)”

read

“II.(1.7.1), II.(1.7.2), II.(1.7.3)”

**page 48**

- line 14, in formula (2.5), for

“ $\mathcal{O}_\ell(1)^{n-1}$ ”

read

“ $\mathcal{O}_\ell(1)^{N-1}$ ”

**page 49**

- line 3, for

“The exact sequence (2.5) yields”

read

“When  $X$  is smooth along  $\ell$  (which certainly happens if the equations defining  $X$  are general), the exact sequence (2.5) yields”

- line 9, for

“ $\alpha(\lambda_2, \dots, \lambda_n)$ ”

read

“ $\alpha(\lambda_2, \dots, \lambda_N)$ ”

**page 50**

- line 9, for

$$\sum_{i \in I_j} x_i^d = 0 \quad \text{for all } j$$

read

$$x_{I_j} \neq 0 \quad \text{and} \quad \sum_{i \in I_j} x_i^d = 0 \quad \text{for all } j$$

**page 51**

- line -1, for

“and the characteristic of  $\mathbf{k}$  is  $\geq d$ .”

read

“and the characteristic of  $\mathbf{k}$  is 0 or  $\geq d$ .”

**page 53**

Replace the beginning of Exercise 3 by

3. (a) If  $p$  is positive and  $d - 1$  is a power of  $p$ , show that there are exactly  $d^3(d - 3)$  nonstandard lines on the surface  $X_3^d$ .

Throughout the rest of this exercise, we assume  $p = 0$  or  $p > d$ .

- (a') Assume  $d \geq 3$ . Show that all lines on  $X_3^d$  are standard (see 2.14). There are therefore  $3d^2$  of them.
- (b) Let  $\ell$  be a standard line contained in  $X_N^d$  which is generic in its  $\mathbf{P}^{r-1}$  (with the notation of 2.14). Show

$$h^0(\ell, N_{\ell/X_N^d}(-1)) = N - 1 - \min(r, d) \quad h^0(\ell, N_{\ell/X_N^d}) = 2N - 2 - \min(2r, d + 1)$$

**page 54**

In Exercise 3.c), add a footnote:

“It was shown by Green (*Some Picard theorems for holomorphic maps to algebraic varieties*, Amer. J. Math. **97** (1975), 43–75) that over the complex numbers, for  $d > n^2 - 1$ , the image of any holomorphic map  $\mathbf{C}^m \rightarrow X_n^d$  is contained in a linear subspace contained in  $X_n^d$  of the type described page 64. In particular, any rational or elliptic curve in  $X_n^d$ , or more generally, any image in  $X_n^d$  of a projective space or a complex torus, is contained in a linear subspace of this form.”

**page 56**

- line -9, for

“1-cycle  $\sum_{i=1}^r (\deg f|_{C_i}) f(C_i)$ .”

read

“1-cycle  $\sum_{i=1}^r d_i f(C_i)$ , where  $d_i$  is the degree of  $f|_{C_i}$  onto its image. Note that for any Cartier divisor  $D$  on  $X$ , one has  $D \cdot f_* C = \deg(f^* D)$ .”

**page 57**

- line 5, delete

“connected”

- line 8, delete

“We may assume that  $C$  is irrational”

- After the illustration, read

“ The 1-cycle  $f_* C$  degenerates to a 1-cycle with a rational component  $e(E)$ .

If  $\text{ev}$  is defined at every point of  $\{c\} \times \bar{T}$ , the rigidity lemma 1.15(a) implies that there exist a neighborhood  $V$  of  $c$  in  $C$  and a factorization

$$\text{ev}|_{V \times \bar{T}} : V \times \bar{T} \xrightarrow{p_1} V \xrightarrow{g} X$$

The morphism  $g$  must be equal to  $f|_V$ . It follows that  $\text{ev}$  and  $f \circ p_1$  coincide on  $V \times T$ , hence on  $C \times T$ . But this means that the image of  $T$  in  $\text{Mor}(C, X; f|_{\{c\}})$  is just the point  $[f]$ , and this is absurd.

Hence there exists a point...”

**page 59**

- After the illustration, for

“ *The rational 1-cycle  $f(C)$  bends and breaks.* ”

read

“ *The 1-cycle  $f_*C$  bends and breaks.* ”

- the text from the last paragraph of the page to the end of the proof should read:

“ Since  $\bar{T}$  is a smooth curve and  $S$  is integral,  $\pi$  is flat ([H1], III, prop. 9.7). Assume that its fibers are all integral. Their genus is then constant ([H1], III, cor. 9.10), hence equal to 0. Therefore, each fiber is a smooth rational curve.

The surface  $S$  is then a (minimal) ruled surface in the sense of [H1], V, §2 (Hartshorne assumes that  $S$  is smooth, but this hypothesis is not used in the proofs hence follows from the others). Let  $T_0$  be the closure of  $\{0\} \times T$  in  $S$  and let  $T_\infty$  be the closure of  $\{\infty\} \times T$ . These sections of  $\pi$  are contracted by  $e$  (to  $f(0)$  and  $f(\infty)$  respectively).

If  $H$  is an ample divisor on  $e(S)$ , which is a surface by construction, we have  $(e^*H)^2 > 0$  and  $e^*H \cdot T_0 = e^*H \cdot T_\infty = 0$ , hence  $T_0^2$  and  $T_\infty^2$  are negative by the Hodge index theorem.

However, since  $T_0$  and  $T_\infty$  are both sections of  $\pi$ , their difference is linearly equivalent to the pull-back by  $\pi$  of a divisor on  $\bar{T}$  ([H1], V, prop. 2.3). In particular,

$$0 = (T_0 - T_\infty)^2 = T_0^2 + T_\infty^2 - 2T_0 \cdot T_\infty < 0$$

which is absurd.

It follows that at least one fiber  $F$  of  $\pi$  is not integral. Since  $S$  is normal, the fibers have no embedded points, hence  $F$  is either reducible or has a multiple component. By Lemma 3.7, the components of  $F$  are all rational curves, and they are not contracted by  $e$ . The direct image of  $F$  on  $X$  is the required 1-cycle.”

- In footnote 1, add:

“ Alternatively, the construction can be made as follows. The indeterminacies of the rational map  $\mathbf{P}^1 \times \bar{T} \dashrightarrow X \times \bar{T}$  induced by  $F$  can be resolved by blowing up points to get a morphism  $\bar{F}' : S' \rightarrow \mathbf{P}^1 \times \bar{T} \dashrightarrow X \times \bar{T}$  whose Stein factorization is  $\bar{F}' : S' \rightarrow S \xrightarrow{\bar{F}} X \times \bar{T}$ . In other words, the surface  $S$  is obtained from  $S'$  by contracting the components of fibers of  $S' \rightarrow \bar{T}$  that are contracted by  $\bar{F}'$ .”

**page 60**

- line -7, for

“if  $D$  is a divisor associated with the line bundle  $\mathcal{O}_Y(1)$ ”

read

“if  $D$  is a divisor associated with the line bundle  $\mathcal{O}_X(1)$ ”

- line -5, for

$$-K_X = (r+1)D + \pi^*(-K_Y - D_1 - \dots - D_r)$$

read

$$-K_X = rD + \pi^*(-K_Y - D_1 - \dots - D_r)$$

**page 61**

- line 2, for

“the relative dualizing sheaf  $\omega_{\mathcal{X}_U/U}$  is ample”

read

“the dual  $\omega_{\mathcal{X}_U/U}^*$  of the relative dualizing sheaf is ample”

**page 62**

- line 25, for

“It follows that  $-K_Y$  is ample”

read

“It follows that  $-K_X$  is ample”

**page 64**

- line 7, for

$$H \cdot \Gamma \leq \frac{2H \cdot C}{\text{Card}(B)}$$

read

$$H \cdot \Gamma \leq \frac{2H \cdot f_*C}{\text{Card}(B)}$$

**page 65**

- After the illustration, for

“ *The curve  $f(C)$  bends and breaks keeping  $c_1, \dots, c_b$  fixed.* ”

read

“ *The 1-cycle  $f_*C$  bends and breaks, keeping  $c_1, \dots, c_b$  fixed.* ”

- line -5, for

“ $N^1(X)_{\mathbf{R}}$ ”

read

“ $N^1(S)_{\mathbf{R}}$ ”

**page 67**

- line 11, for

$$H \cdot \Gamma \leq 2 \dim(X) \frac{H \cdot C}{-K_X \cdot C}$$

read

$$H \cdot \Gamma \leq 2 \dim(X) \frac{H \cdot f_*C}{-K_X \cdot f_*C}$$

- lines -7 and -5, for

$$-p^m K_X \cdot C$$

read

$$-p^m K_X \cdot f_* C$$

- line -3, for  
“a rational curve  $\Gamma_m$ ”

read

“a rational curve  $\Gamma_m$  on  $X$ ”

- line -1, for

$$H \cdot \Gamma_m \leq \frac{2H \cdot C_m}{b_m} = \frac{2p^m}{b_m} (H \cdot C)$$

read

$$H \cdot \Gamma_m \leq \frac{2H \cdot (f_m)_* C_m}{b_m} = \frac{2p^m}{b_m} H \cdot f_* C$$

**page 68**

- line 1, for  
“ $-\dim(X)/(-K_X \cdot C)$ .”

read

“ $-\dim(X)/(-K_X \cdot f_* C)$ .”

- line 3, for

$$H \cdot \Gamma_m \leq \frac{2 \dim(X)}{-K_X \cdot C} (H \cdot C)$$

read

$$H \cdot \Gamma_m \leq \frac{2 \dim(X)}{-K_X \cdot f_* C} H \cdot f_* C$$

- line 13, for  
“of degree at most  $2 \dim(X) \frac{H \cdot C}{-K_X \cdot C}$ ”

read

“of degree at most  $2 \dim(X) \frac{H \cdot f_* C}{-K_X \cdot f_* C}$ ”

- line 17, for

“Let  $M_d$  be the quasi-projective scheme that parametrizes rational curves on  $X$ ”

read

“Let  $M_d$  be the quasi-projective scheme that parametrizes morphisms  $\mathbf{P}^1 \rightarrow X$ ”

**page 71**

- line 1, delete

“connected”

- After the illustration, for

“ *The curve  $f(C)$  bends and breaks: ...* ”

read

“ *The 1-cycle  $f_*C$  bends and breaks: ...* ”

**page 77**

- line 2, for

“any such morphism factors through  $\alpha_X$ .”

read

“any such morphism factors uniquely through  $\alpha_X$ .”

**page 79**

- line -4, for

“ $\pi^{-1}(\pi(x))$ ”

read

“ $\pi^{-1}(\pi(f(c)))$ ”

**page 80**

- After the illustration, for

“*The curve  $f(C)$  bends and breaks: a non vertical rational 1-cycle  $e(E)$  appears.*”

read

“ *The 1-cycle  $f_*C$  bends and breaks: a nonvertical 1-cycle  $e_*E$  appears.* ”

**page 81**

- line -12, delete

“connected”

**page 83**

- line -10, delete

“connected”

**page 88**

- line 4, for

“It follows that the map  $\text{ev}_d^K$  is also dominant.”

read

“It follows that the map  $\text{ev}_d^k$  is also dominant.”

- line 14, for

“the morphism  $\text{Mor}_d(\mathbf{P}^1, \tilde{X}) \rightarrow \text{Mor}_d(\mathbf{P}^1, X)$ ”

read

“the morphism  $\text{Mor}(\mathbf{P}^1, \tilde{X}) \rightarrow \text{Mor}(\mathbf{P}^1, X)$ ”

**page 89**

- lines 1 and 16, for

“for some  $d$ ”

read

“for some positive  $d$ ”

- line 17, for

“the morphism  $\text{Mor}_d(\mathbf{P}^1, \tilde{X}) \rightarrow \text{Mor}_d(\mathbf{P}^1, X)$ ”

read

“the morphism  $\text{Mor}(\mathbf{P}^1, \tilde{X}) \rightarrow \text{Mor}(\mathbf{P}^1, X)$ ”

**page 90**

- line 12, for

“ With our notation, this means  $a_n \geq r$ . We will say”

read

“ With our notation, this means  $a_n \geq r$ . Note that the composition of an  $r$ -free morphism with any automorphism of  $\mathbf{P}^1$  is still  $r$ -free. We will say that a rational curve on  $X$  is  $r$ -free if its normalization is.

We will say”

- line 22, for

“by Theorem 2.6.”

read

“by Theorem 2.6. and its extension 2.9.”

- line -8, for

“there is an exact sequence”

read

“there is an exact sequence ([H1], II, Theorem 8.17)”

**page 94**

- line -7, for

“Let  $\text{ev}_i : \mathbf{P}^1 \times M_i \rightarrow X$  be the evaluation maps.”

read

“Let  $e_i : \mathbf{P}^1 \times (M_i)_{\text{red}} \rightarrow X$  be the morphisms induced by the evaluation maps.”

and replace  $\text{ev}_i$  by  $e_i$  in the rest of the proof.

- line -3, for

“[H1], III, prop. 4.6”

read

“[H1], III, Corollary 10.7”

**page 97**

- line 15, for

“[H1], III, prop. 4.6”

read

“[H1], III, Corollary 10.7”

**page 98**

- line 11, for

“the injection  $\iota : \{0\} \times M \hookrightarrow X$ ”

read

“the injection  $\iota : \{0\} \times M \hookrightarrow \mathbf{P}^1 \times M$ ”

- line 13, for

$$\pi_1(\iota) \circ \pi_1(\text{ev}) = 0$$

read

$$\pi_1(\text{ev}) \circ \pi_1(\iota) = 0$$

- line -15, for

“Let  $\text{ev}_i : \mathbf{P}^1 \times M_i \rightarrow X$  be the evaluation maps.”

read

“Let  $e_i : \mathbf{P}^1 \times (M_i)_{\text{red}} \rightarrow X$  be the morphisms induced by the evaluation maps.”

and replace  $\text{ev}_i$  by  $e_i$  in the rest of the proof.

- line -10, for

“ $X^{\text{free}}$ ”

read

“ $X_x^{\text{free}}$ ”

**page 100**

- in footnote 4, for

“The proof follows the arguments used for uniruled or rationally connected varieties, using the closed subscheme  $\text{Rat}(X)$  of  $\text{Hilb}(X)$  that will be defined in 5.6, instead of  $\text{Mor}(\mathbf{P}^1, X)$ .”

read

“When  $X$  is projective, the proof follows the arguments used for uniruled or rationally connected varieties, using the closed subscheme  $\text{Rat}(X)$  of  $\text{Hilb}(X)$  that will be defined in 5.6, instead of  $\text{Mor}(\mathbf{P}^1, X)$ . For the general case, see [K1], prop. IV.3.6.”

**page 114**

- line -15, for  
“an irreducible constructible subset”  
read

- “a constructible subset”

- from line -12 on, read

“The right-hand side is contained in the left-hand side. Let us prove the other inclusion. We may assume by Lemma 5.1 that  $V$  is irreducible. By [H1], II, ex. 3.18(b),  $V$  contains a subset  $U$  that is dense open in  $\bar{V}$ . Let  $W$  be a component of  $\bar{V} - U$ . By a theorem of Chevalley ([H1], II, ex. 3.22(e)), a general fiber of  $W \rightarrow \pi(\bar{V})$  is either empty or everywhere of dimension

$$\dim(W) - \dim(\pi(\bar{V}))$$

which is less than the dimension  $\dim(\bar{V}) - \dim(\pi(\bar{V}))$  of the fiber of  $\bar{V} \rightarrow \pi(\bar{V})$  at the same point. It follows that for  $y$  general in  $\pi(\bar{V})$ , the closed subset  $(\bar{V} - U) \cap \pi^{-1}(y)$  of  $\bar{V} \cap \pi^{-1}(y)$  is nowhere dense, hence that  $U \cap \pi^{-1}(y)$  is dense in  $\bar{V} \cap \pi^{-1}(y)$ .”

#### page 116

- line 8, for  
“separable closure of  $K$  and let”  
read  
“separable closure of  $K$ , and let”

#### page 117

- line -5, for  
“of length  $m$ ”  
read  
“of length 1”

#### page 118

- line 5, for  
“ $\overline{F(W_m^{i,j})}$ ”

read  
“ $\overline{F(W_m^{i,j})}$ ”

- line -12 on, read

“ Call  $\overline{V_m^i}$  *stable* if  $\overline{F(\tilde{W}_m^{i,j})} = \overline{V_m^i}$  for all  $j$ , *unstable* otherwise. Note the following:

- if all components of  $\overline{V_m(x)}$  are stable,  $\overline{V_{m+1}(x)} = \overline{V_m(x)}$ ;
- if  $\overline{V_m^i}$  is unstable, it is strictly contained in an irreducible component of  $\overline{V_{m+1}(x)}$ ;
- if  $\overline{V_m^i}$  is stable and an irreducible component of  $\overline{V_{m+1}(x)}$ , it is stable as a component of  $\overline{V_{m+1}(x)}$ .

Let  $\overline{V_m^0} = \overline{F(\overline{W_{m-1}^{i,j}})}$  be an unstable component of  $\overline{V_m(x)}$ . If the corresponding  $\overline{V_{m-1}^i}$  is a stable component of  $\overline{V_{m-1}(x)}$ , it is equal to  $\overline{V_m^0}$  by definition, which contradicts the third item. Hence  $\overline{V_{m-1}^i}$  is unstable, and is strictly contained in  $\overline{V_m^0}$  by the second item.

It follows that if  $\overline{V_m(x)}$  has an unstable component  $\overline{V_m^0}$ , then  $\overline{V_{m-1}(x)}$  also has an unstable component of smaller dimension. In particular, the dimension of  $\overline{V_m^0}$  is at least  $m$ . By the second item, this implies  $\dim(\overline{V_{m+1}(x)}) > m$ , which is impossible since  $m \geq \delta(x)$ . Therefore,  $\overline{V_m(x)}$  has only stable components, and  $\overline{V_{m+1}(x)} = \overline{V_m(x)}$  by the first item. This proves the first step.”

**page 119**

- line 5, for

$$V = \bigcup_{x \in X} (\{x\} \times V_n(x))$$

read

$$V = \bigcup_{x \in X} (V_n(x) \times \{x\})$$

**page 123**

- line -2, for

“hence factors as  $X'^* \rightarrow X^* \xrightarrow{\rho} Y$ ,”

read

“hence factors as  $X'^* \rightarrow X^* \xrightarrow{\rho} Y^*$ ,”

**page 131**

- line -15, for

“Let  $\text{Rat}_{n+1}(X)$  be the closure in  $\text{Hilb}(X)$  of the scheme that parametrizes curves”

read

“Let  $\text{Rat}_{n+1}(X)$  be the (projective) subscheme of  $\text{Hilb}(X)$  that parametrizes curves”

**page 133**

- line 7, delete

“Examples in Section 5.11 show that  $k$  may grow as  $\log n$ .”

**page 135**

- line -9, for

“8 points”

read

“8 points in general position”

- line -6, for  
“87 other families with Picard number  $> 1$  ([MM]).”

read

“88 other families with Picard number  $> 1$  (Mori and Mukai recently noticed that the family of blow-ups of  $\mathbf{P}^1 \times \mathbf{P}^1 \times \mathbf{P}^1$  along a curve of tridegree  $(1, 1, 3)$  is missing from the list in [MM]).”

**page 136**

- line -18, for  
“the curve  $C_1 = f^{-1}(C)$ .”

read

“the curve  $C_1 = \pi^{-1}(C)$ .”

**pages 136 and 137**

- delete footnote 15 and replace footnote 16 with

“ In this direction, complex  $\mathbf{Q}$ -Fano threefolds with canonical singularities (see Definition 7.13) and Picard number 1 are known to form a limited family ([Ka4] for the case of terminal singularities; the canonical case is in Kollár, J., Miyaoka, Y., Mori, S. and Takagi, H., Boundedness of canonical  $\mathbf{Q}$ -Fano 3-folds, *Proc. Japan Acad. Ser. A Math. Sci.* **76** (2000), 73–77). The point is again to bound  $(-K_X)^n$  and an integer  $j$  such that  $jK_X$  is a Cartier divisor. Work of Ran and Clemens ([RC]) shows that  $\mathbf{Q}$ -Fano  $n$ -folds with canonical singularities and Picard number 1 form a limited family if one moreover bounds the smallest number  $m$  such that  $-mK_X$  is very ample (in the smooth case, one can take  $m = n(n+1)(n+3)$  as in the proof of Theorem 5.19).

More generally, it is conjectured that given  $\varepsilon > 0$ , Fano varieties  $X$  such that, for any desingularization  $f : \tilde{X} \rightarrow X$ , one has  $K_{\tilde{X}} = f^*K_X + \sum a_i E_i$  with  $a_i > -1 + \varepsilon$ , where the  $E_i$  are the exceptional divisors of  $f$ , still form a limited family. Our surfaces fall in this class for  $g = 0$  and  $d < 2/\varepsilon$ .”

**page 138**

- line 2, read,

“or by noting that  $L$  and  $H$  are nef, and  $L + H$  is ample, since its associated line bundle is the tautological line bundle associated with the description of  $X$  as  $\mathbf{P}(\mathcal{O}_{\mathbf{P}^s}^r(1) \oplus \mathcal{O}_{\mathbf{P}^s}^r(a+1))$ ; it is therefore ample.”

- replace lines 11 through 15 with

Setting  $n = \dim(X) = r + s$ , we get, since  $L$  and  $H$  are nef, for  $a = s$

$$(-K_X)^n \geq ((r+1)L)^n = (r+1)^n a^s L^r \cdot H^s = (r+1)^n a^s$$

- line -1, for  
 $H^0(\mathbf{P}^s, \mathcal{O}_{\mathbf{P}^s}(s-r))^r$

read,

$$H^0(\mathbf{P}^s, \mathcal{O}_{\mathbf{P}^s}(s))^r$$

**page 141**

- line 18, for  
“yields the (5.3).”  
read  
“yields (5.3).”

**page 146**

- line 10, for  
“ the Riemann-Roch theorem yields

$$h^0(X, \mathcal{O}_X(mC)) = m + \chi(X, \mathcal{O}_X) \geq 2$$

and the linear system  $|mC|$  has no base-point (the only possible fixed curve is  $C$ , but  $h^0(X, \mathcal{O}_X((m-1)C)) < h^0(X, \mathcal{O}_X(mC))$ , and there are no isolated base-points since  $C^2 = 0$ .)”

- read  
“ the Riemann-Roch theorem yields

$$h^0(X, \mathcal{O}_X(mC)) \geq m + \chi(X, \mathcal{O}_X) \geq 2$$

Since the only possible fixed curve in the linear system  $|mC|$  is  $C$ , the moving part of the linear system  $|mC|$  is of the form  $|m'C|$  for some nonnegative integer  $m' \leq m$ . Since  $(m'C)^2 = 0$ , the linear system  $|m'C|$  has no base-points.”

**page 147**

- line -2, for  
“Lemma 6.2(d)”  
read  
“Lemma 6.2(e)”

**page 149**

- l.14, for  
“and  $E$  the last  $(-1)$ -curve of the composition of blow-ups  $\varepsilon$ , its image in  $X$  is a curve,”  
read  
“and if  $E$  is the last  $(-1)$ -curve of the composition of blow-ups  $\varepsilon$ , its image  $\bar{\sigma}(E)$  in  $X$  is a curve,”  
• l.18, for  
“there are no exceptional curves in  $\tilde{X}$ ,”  
read  
“there are no such curves  $E$  in  $\tilde{X}$ ,”

**page 152**

- line 3, for

$$H \cdot \Gamma_i < \varepsilon^{-1} K_X \cdot \Gamma_i \leq \varepsilon^{-1} (\dim(X) + 1)$$

read

$$H \cdot \Gamma_i < -\frac{1}{\varepsilon} K_X \cdot \Gamma_i \leq \frac{1}{\varepsilon} (\dim(X) + 1)$$

- line -5, starting from “Let  $V_J$  be this cone...” replace the end of the proof with

“ By Lemma 6.7(b), it is enough to show that any extremal ray  $\mathbf{R}^+r$  in  $\overline{V_J}$  satisfying  $K_X \cdot r < 0$  is in  $V_J$ . Let  $H$  be an ample divisor on  $X$  and let  $\varepsilon$  be a positive number such that  $(K_X + \varepsilon H) \cdot r < 0$ . By the first step, there are only finitely many classes  $z_{j_1}, \dots, z_{j_q}$ , with  $j_\alpha \in J$ , such that  $(K_X + \varepsilon H) \cdot z_{j_\alpha} < 0$ .

Write  $r$  as the limit of a sequence  $(r_m + s_m)$ , where  $r_m \in \overline{\text{NE}}(X)_{K_X + \varepsilon H \geq 0}$  and  $s_m = \sum_{\alpha=1}^q l_{\alpha,m} z_{j_\alpha}$ . Since  $H \cdot r_m$  and  $H \cdot z_{j_\alpha}$  are positive, the sequences  $(H \cdot r_m)$  and  $(l_{\alpha,m})$  are bounded, hence we may assume, after taking subsequences, that all sequences  $(r_m)$  and  $(l_{\alpha,m})$  have limits (Theorem 1.27(b)). Because  $r$  spans an extremal ray in  $\overline{V_J}$ , the limits must be nonnegative multiples of  $r$ , and since  $(K_X + \varepsilon H) \cdot r < 0$ , the limit of  $(r_m)$  must vanish. Moreover,  $r$  is a multiple of one the  $z_{j_\alpha}$ , hence is in  $V_J$ .

If we choose a set  $I$  of indices such that  $(\mathbf{R}^+z_j)_{j \in I}$  is the set of all (distinct) extremal rays among all  $\mathbf{R}^+z_i$ , the proof shows that any extremal ray of  $\overline{\text{NE}}(X)_{K_X < 0}$  is spanned by a  $z_i$ , with  $i \in I$ . This finishes the proof of the cone theorem.  $\square$ ”

**page 154**

- line -14, for

“component of  $R$ ”

read

“component of the locus of  $R$ ”

**page 157**

- lines 31–32, read

“

$$K_X = -r\xi + \pi^*(K_Y + \det(\mathcal{E}))$$

If  $\ell$  is the class of a line contained in a fiber of  $\pi$ , we have  $K_X \cdot \ell = -r$ .”

**page 158**

- lines 19 and 24, for

“ $d > 2g - 1$ ”

read

“ $d \geq 2g - 1$ ”

**page 160**

- footnote 5: line -7, for

“Its blow-up is the total space of the  $\mathbf{P}^1$ -bundle  $\mathbf{P}(\mathcal{O}_Q \oplus \mathcal{O}_Q(1)) \rightarrow Q$ , where  $Q$  is a smooth quadric in  $\mathbf{P}^3$ , and the exceptional divisor is the image of the section corresponding to the trivial quotient of  $\mathcal{O}_Q \oplus \mathcal{O}_Q(1)$ .”

read

“If  $Q$  is a smooth quadric in  $\mathbf{P}^3$ , and  $Q_0$  and  $Q_\infty$  are the two sections of the  $\mathbf{P}^1$ -bundle  $\mathbf{P}(\mathcal{O}_Q \oplus \mathcal{O}_Q(1)) \rightarrow Q$  corresponding to the quotients  $\mathcal{O}_Q$  and  $\mathcal{O}_Q(1)$  of  $\mathcal{O}_Q \oplus \mathcal{O}_Q(1)$ , its blow-up is  $\mathbf{P}(\mathcal{O}_Q \oplus \mathcal{O}_Q(1)) - Q_\infty$  and the exceptional divisor is  $Q_0$ .”

**page 161**

- footnote 7: line -6, read

$$\left(1 - a_{22} - \frac{a_{12}a_{21}}{1 - a_{11}}\right)[L_2] = \frac{a_{21}}{1 - a_{11}}(r_1 + z_1) + (r_2 + z_2)$$

**page 162**

- line 30, read

$$S_1^+ = \mathbf{P}(N_{\Gamma^+/X^+}) = \mathbf{P}(\mathcal{O}_{\mathbf{P}^1} \oplus \mathcal{O}_{\mathbf{P}^1}(1))$$

- footnote 9: line -3, for  
“with exceptional divisor  $S = \mathbf{P}(N_{C/Y})$ .”  
read  
“with exceptional divisor  $S = \mathbf{P}(N_{C/Y}^*)$ .”

**page 163**

- line 13, for  
“relation  $0 = -a + b$ , which is absurd.”  
read  
“relation  $-1 = -a + b$ , which is absurd.”
- footnote 9: line -1, for  
“which is split if  $\deg(N_{C/Y}) \geq 3C_0^2 - 2$ .”  
read  
“which is split if  $\deg(N_{C/Y}) \leq 3C_0^2 + 2$ .”

**page 165**

- line 8, for  
“let  $M$  be a nef divisor”  
read  
“let  $M$  be an ample divisor”

**page 179**

- line 1, for

“with poles at every exceptional divisor”

read

“that vanishes along every exceptional divisor”

**page 181**

- line 16, for

“let  $-D$  be an”

read

“let  $-D$  be a”

**page 184**

- footnote 8: for

“There are now simple proofs of this statement (see [AJ], [BP]).”

read

“There are now simple proofs of this statement (see [AJ], [BP], and especially K. H. Paranjape, *The Bogomolov–Pantev resolution, an expository account*, New trends in algebraic geometry (Warwick, 1996), Cambridge University Press, Cambridge, 1999, pp. 347–358).”

**page 189**

- line 15, for

$$H^0(X, m\pi^*D + E)$$

read

$$H^0(Y, m\pi^*D + E)$$

**page 191**

- Invoking Riemann-Roch in the first step of the proof of the theorem is a bit overkill. The following more elementary statement, more in the spirit of Chapter 1 of this book, is sufficient for our purpose here. Its proof can be found in [Kle], §2, Theorem 1, or Theorem 9.6.3 of the more recent reference: S. Kleiman, The Picard scheme. *Fundamental algebraic geometry*, 235–321, Math. Surveys Monogr. **123**, Amer. Math. Soc., Providence, RI, 2005.

*Let  $X$  a proper scheme of dimension  $n$  over a field and let  $D$  be a Cartier divisor on  $X$ . The following properties are equivalent:*

- (i) *the divisor  $D$  is numerically equivalent to 0;*
- (ii) *for any coherent sheaf  $\mathcal{F}$  on  $X$ , we have  $\chi(X, \mathcal{F}(D)) = \chi(X, \mathcal{F})$ ;*
- (iii) *for all Cartier divisors  $D_1, \dots, D_{n-1}$  on  $X$ , we have  $(D \cdot D_1 \cdot \dots \cdot D_{n-1}) = 0$ ;*
- (iv) *for any Cartier divisor  $E$  on  $X$ , we have  $(D \cdot E^{n-1}) = 0$ .*

(The only difficult implication is (i)  $\Rightarrow$  (ii).)

**page 195**

- line 25, for  
“let  $B(b)$  be the base-locus”  
read  
“let  $B(b)$  be the (reduced) base-locus”

**page 196**

- lines 9–12, read  
“(b)  $K_Y \sim \pi^*(K_X + \Delta) + \sum_i a_i F_i$ , with  $a_i > -1$  for all  $i$ ;  
(c) the divisor  $\pi^*(aD - (K_X + \Delta)) - \sum_i p_i F_i$  is ample, where  $p_i \in (0, 1 + a_i)$  for all  $i$ .”  
• line -10, for “property (b)”, read “property (c)”

**page 197**

- line -7, for “rays).” read “rays.”  
• line -1, for “(1.29).” read “(1.29).”

**page 199**

- line -14, for  
“Let  $B(p, q)$  be its base-locus.”  
read  
“Let  $B(p, q)$  be its (reduced) base-locus.”

**page 200**

- line -9 on, read  
“There exists as in the proof of the base-point-free theorem 7.32 a desingularization  $\pi : Y \rightarrow X$  and divisors  $F_i$  on  $Y$  (not necessarily exceptional) such that  
(a) the linear system  $|\pi^*(p_0 H + q_0 j_{X, \Delta}(K_X + \Delta)) - \sum_i r_i F_i|$  is base-point-free, where  $r_i \geq 0$  for all  $i$ ;  
(b)  $K_Y \sim \pi^*(K_X + \Delta) + \sum_i a_i F_i$ , with  $a_i > -1$  for all  $i$ ;  
(c) the divisor  $\pi^*(p_0 H + (q_0 j_{X, \Delta} - 1)(K_X + \Delta)) - \sum_i p_i F_i$  is ample, where  $p_i \in (0, 1 + a_i)$  for all  $i$ .”

**page 201**

- line 13, for “ $aq_0$ ” read “ $j_{X, \Delta} q_0$ ”  
• line -11, for “ $-(K_X + \Delta)$ ” read “ $-\pi^*(K_X + \Delta)$ ”

**page 202**

- line 10, for  
“the base-locus of the left-hand side”  
read

“the (reduced) base-locus of the left-hand side”

page 203

- line 26, for

$$\frac{u_m}{v} = \sup\{t \in \mathbf{R} \mid mM + H + t(K_X + \Delta) \text{ nef}\}$$

read

$$\frac{j_{X,\Delta}u_m}{v} = \sup\{t \in \mathbf{R} \mid mM + H + t(K_X + \Delta) \text{ nef}\}$$

- line 29, for

“( $mM + H + \frac{u_m}{v}(K_X + \Delta) \cdot z_0 \geq 0$  ”

read

“( $mM + H + \frac{j_{X,\Delta}u_m}{v}(K_X + \Delta) \cdot z_0 \geq 0$  ”

- line 30, for

$$u_m \leq v \frac{(mM + H) \cdot z_0}{-(K_X + \Delta) \cdot z_0} = v \frac{H \cdot z_0}{-(K_X + \Delta) \cdot z_0}$$

read

$$j_{X,\Delta}u_m \leq v \frac{(mM + H) \cdot z_0}{-(K_X + \Delta) \cdot z_0} = v \frac{H \cdot z_0}{-(K_X + \Delta) \cdot z_0}$$

- line 33, for

$$L_m = v(mM + H + \frac{u_\infty}{v}(K_X + \Delta))$$

read

$$L_m = v(mM + H + \frac{j_{X,\Delta}u_\infty}{v}(K_X + \Delta))$$

page 204

- line 8, for

$$(L_H)|_{V_M} = (H + \frac{u_\infty}{v}(K_X + \Delta))|_{V_M}$$

read

$$(L_H)|_{V_M} = (H + \frac{j_{X,\Delta}u_\infty}{v}(K_X + \Delta))|_{V_M}$$

- line 11, for

“Second step:  $\overline{\text{NE}}(X)$  is the closure of

$$V = \overline{\text{NE}}(X)_{K_X + \Delta \geq 0} + \sum_{L \in \mathcal{L}} V_L$$

where  $\mathcal{L}$  is the set of nef nonample divisors  $L$  such that  $V_L$  is a  $(K_X + \Delta)$ -negative extremal ray.

By Lemma 6.7(a), the boundary of the dual cone  $\overline{\text{NE}}(X)^*$  meets the interior of  $\overline{V}^*$  at a point that corresponds to a nonample nef  $\mathbf{R}$ -divisor  $M$  positive on  $\overline{V} - \{0\}$ .”

read

“Second step:  $\overline{\text{NE}}(X)$  is the closure of

$$V = \overline{\text{NE}}(X)_{K_X + \Delta \geq 0} + \sum_{V_L \in \mathcal{L}} V_L$$

where  $\mathcal{L}$  is the set of  $(K_X + \Delta)$ -negative extremal rays of the type  $V_L$ , with  $L$  nef nonample divisor.

If this is not the case, there exists by Lemma 6.7(d) (since  $\overline{\text{NE}}(X)$  contains no lines) an  $\mathbf{R}$ -divisor  $M$  on  $X$  which is nonnegative on  $\overline{\text{NE}}(X)$  but vanishes on some extremal ray (it is in particular nef and nonample), and is positive on  $\overline{V} - \{0\}$ .”

• line -4, for

“Third step:  $\mathcal{L}$  is countable and the  $(K_X + \Delta)$ -negative rays  $(V_L)_{L \in \mathcal{L}}$  are locally discrete in the half-space  $N_1(X)_{K_X + \Delta < 0}$ .”

read

“Third step:  $\mathcal{L}$  is countable and locally discrete in the half-space  $N_1(X)_{K_X + \Delta < 0}$ .”

**page 205**

• line 1, for

“For any  $L \in \mathcal{L}$ ,”

read

“For any  $V_L \in \mathcal{L}$ ,”

• line 6, for

“if there are infinitely rays  $V_L$ ,”

read

“if there are infinitely many rays  $V_L$  in that half-space,”

• line 8, for

“Fourth step: for any subset  $\mathcal{L}'$  of  $\mathcal{L}$ , the cone

$$\overline{\text{NE}}(X)_{K_X + \Delta \geq 0} + \sum_{L \in \mathcal{L}'} V_L$$

is closed.”

read

“Fourth step: for any subset  $\mathcal{L}'$  of  $\mathcal{L}$ , the cone

$$\overline{\text{NE}}(X)_{K_X + \Delta \geq 0} + \sum_{V_L \in \mathcal{L}'} V_L$$

is closed.”

**page 211**

- line 7, for

“If  $E$  is normal, we can proceed as in 7.45. Singularities of  $E$  will be our main concern.

Since  $Y$  is normal,  $c(E)$  has codimension at least 2 in  $Y$  and  $E = c^{-1}(c(E))$  (1.40).”

read

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Since  $Y$  is normal,  $c(E)$  has codimension at least 2 in  $Y$  and  $E = c^{-1}(c(E))$  (1.40).”

read

“Replacing  $c$  with its Stein factorization, we may assume that it has connected fibers. Since  $Y$  is normal,  $E = c^{-1}(c(E))$  (1.40).”

- line 13, for

“the  $c$ -ampleness of  $-K_X$ ”

read

“the  $c$ -ampleness of  $-(K_X + \Delta)$ ”

**page 212**

- line -17, for “exact sequences” read “spectral sequences”

**page 219**

- after Exercise 8, add a new exercise:

Let  $X$  be a smooth projective variety of dimension  $n$  such that  $(-K_X)^n > 0$  and  $K_X \cdot C < 0$  for every curve  $C$  on  $X$ . Show that  $X$  is a Fano variety. (*Hint*: use the base-point-free theorem 7.32.)