Homework 2: Second Order ODE

Due: 10/16/2021

Name: *Univ ID:*

Throughout the exercises, we always use $y(t)$ to represent the unknown function and t for variable.

Exercise 1. *(4 points) Find the general solution for the following equations*

1. $y'' + 4iy = 0$;

2. $y'' - 4y' + 6y = 0$.

Exercise 2. *(4 points) Find the general solution for the following equations*

1. $y'' + 5y' + 4y = e^{4t}$;

2. $y'' - 9y = 2t^2 - 5$.

Exercise 3. *(3 points) Find the general solution for the equation*

$$t^2 y'' + t y' - y = 0.$$

(Hint: one solution of this equation is $y(t) = t$.)

Exercise 4. *(3 points) Find the solution for $y'' + 2y' - 15y = 0, y(0) = 1, y'(0) = 2$.*

Exercise 5 (Harmonic Oscillator)**.** *(6 points) A harmonic oscillator is an object experiencing a restoring force F_H proportional to the displacement y,*

$$F_H = -ky,$$

and friction F_F that opposes motion, proportional to the velocity y'

$$F_F = -cy'.$$

1. *Write down the second order ODE of the harmonic oscillator using Newton's second law.*

2. *Write down its associated characteristic equation.*

3. *Give its general solution and describe the behavior for different cases. (Hint: its characteristic equation can have two real roots, two equal real roots or two two complex roots.)*

4. *(Bonus 2 points) Suppose now the friction is 0, but there is an external force $F(t) = F_0 cos(\omega t)$. Use this equation to explain the phenomena of resonance in physic.*