
(2.10) Proposition. Assume that there are tuples $a$ and $b$, and a model $M$ of $T$, such that $tp_T(b/M, a)$ is a heir of $tp_T(b/M)$, and $acl_T(M, a, b) \cap S \not\subseteq acl_T(M, a) \cup acl_T(M, b)$. Then $T_{P, S}$ has the independence property.

Proof. We choose an indiscernible sequence $(b_i), i \in \mathbb{N}$, of realisations of $tp_T(b/M, a)$, such that for every $i \in \mathbb{N}$, $tp_T(b_i/M, a, b_0, \ldots, b_{i-1})$ is the heir of $tp_T(b/M)$. Let $\varphi(x, a, b)$ be an $L(M)$-formula isolating the type over $(M, a, b)$ of an element $\alpha \in acl_T(M, a, b) \cap S$, $\alpha \not\in acl_T(M, a) \cup acl_T(M, b)$. Then, for any $i$ the elements satisfying $\varphi(x, a, b_i)$ are not in $acl_T(M, a, b_0, \ldots, b_{i-1})$ nor in $acl_T(M, b_j, j \in \mathbb{N})$. Let $I$ be a subset of $\mathbb{N}$, and let $T'$ be a completion of $T_{P, S}$. Consider the subset $P^N$ of $N = acl_T(M, a, b_j, j \in \mathbb{N})$ defined as follows: $T'$ tells us which elements of $acl_T(\emptyset)$ must be in $P$, and we let $P^N \cap acl_T(\emptyset)$ be this set. If $x \not\in acl_T(\emptyset)$, then $x \in P^N$ if and only if $x$ satisfies $\varphi(x, a, b_i)$ for some $i \in I$. Then $(N, P^N)$ embeds in a model of $T'$, and in this model the sequence $(b_i), i \in \mathbb{N}$, is indiscernible. This shows that $T'$ has the independence property, by ??? of [Po].

(3.7). Add at the beginning of the proof: moving $\bar{c}_1$ by an $E$-automorphism, we may assume that $tp_T(\bar{c}_1/E, \bar{a}, \bar{b})$ does not fork over $E$.

(3.10) Proposition. Assume that there is a model $M$ of $T$, and tuples $a$ and $b$ which are independent over $M$ and such that $acl_T(M, a, b) \neq dcl_T(acl_T(M, a), acl_T(M, b))$. Then $T_A$ has the independence property.

Proof. The proof begins as in the paper. One needs however to make sure that the sequence $b_i, i \in \mathbb{N}$, is indiscernible in the sense of $T_A$. We are working in a big model $M^*$ of $T$ containing everything. We define $\sigma$ to be the identity on $A$ and on $acl_T(M, b_i, i \in \mathbb{N})$. Then the sequence $(b_i), i \in \mathbb{N}$, will be indiscernible in any model of $T_A$ containing $(acl_T(M, b_i, i \in \mathbb{N}), \sigma)$. Let $C$ be the set obtained by adjoining to $A \cup acl_T(M, b_i, i \in \mathbb{N})$ the elements satisfying $\varphi(x, a, b_i)$ for some $i \in \mathbb{N}$. Extend $\sigma$ to an elementary (in the sense of $N$) permutation of $C$ by imposing that $\sigma$ is the identity on the set of elements satisfying $\varphi(x, a, b_i)$ if and only if $i \in I$. Then $(C, \sigma)$ embeds in a model of $T_A$. 

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