Screened Poisson Surface Reconstruction

Jean Feydy

École Normale Supérieure

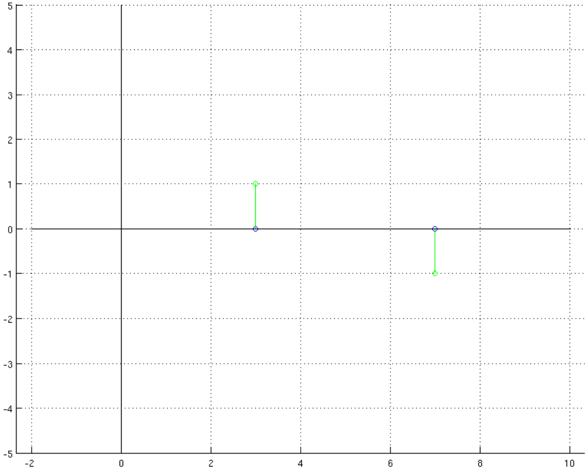
23 mars 2015

Kazhdan, M., Bolitho, M., and Hoppe, H. (2006).
Poisson surface reconstruction.
In *Proceedings of the fourth Eurographics symposium on Geometry processing*, volume 7.

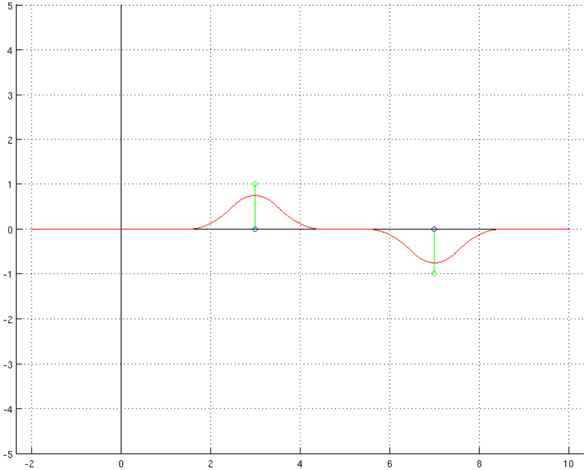
Kazhdan, M. and Hoppe, H. (2013).
Screened poisson surface reconstruction.
ACM Transactions on Graphics (TOG), 32(3) :29.

Présentation de la méthode de Poisson écrantée

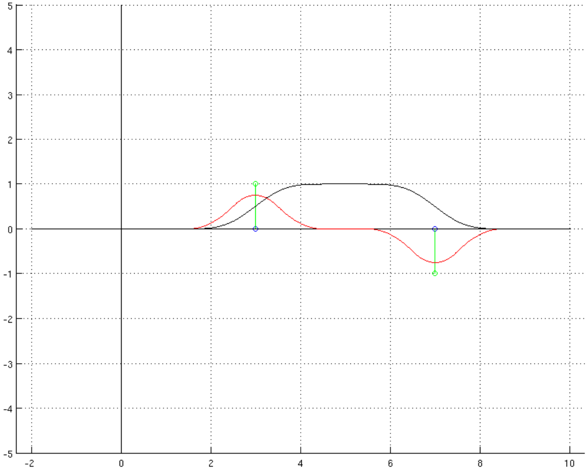
P, N_p



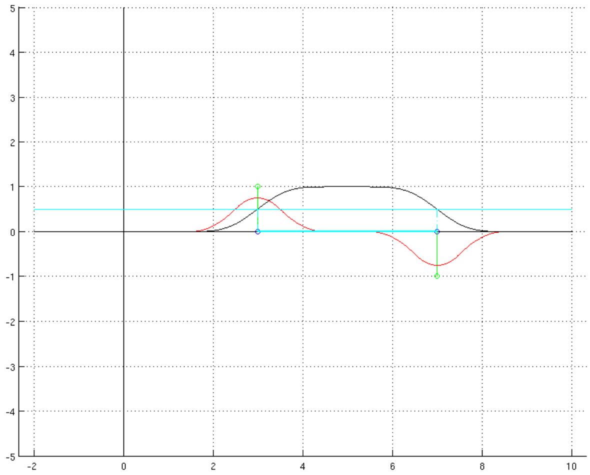
P, N_p, $V = N_p \star F$



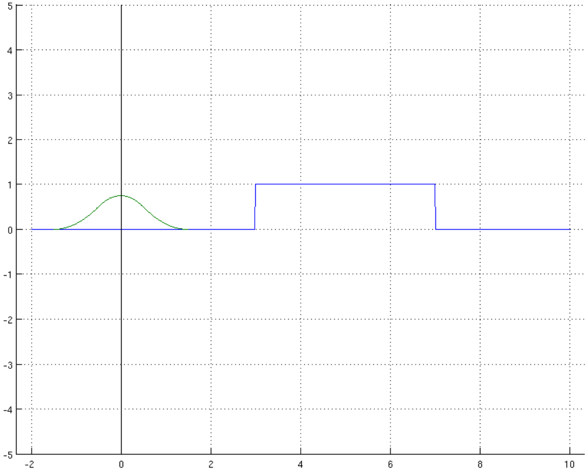
P, N_p, $V = N_p \star F$, $\chi = \int V$,



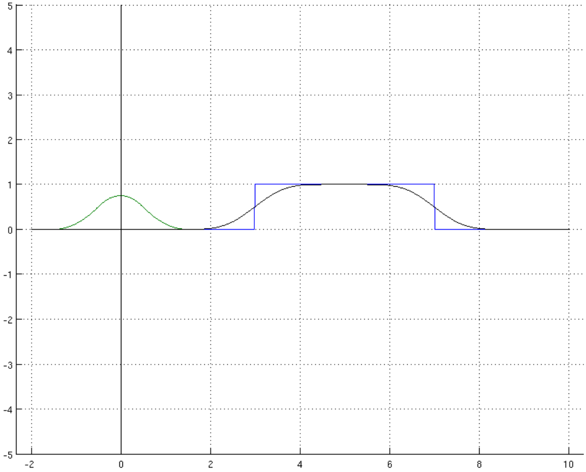
P, N_p, $V = N_p \star F$, $\chi = \int V$, $\{\chi \geqslant 1/2\}$



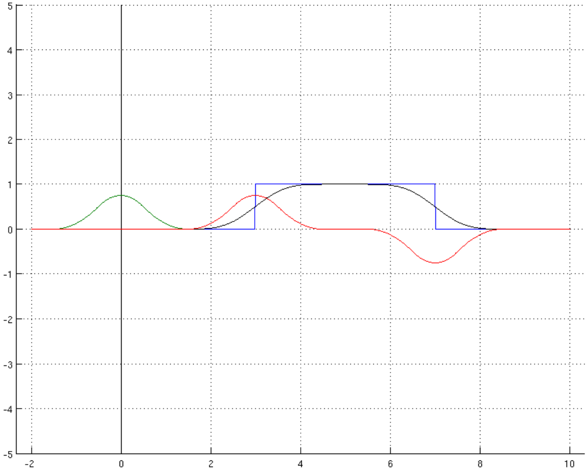
$\mathbb{1}_V$, F



$\mathbb{1}_V$, F, $\mathbb{1}_V \star F$



$\mathbb{1}_V$, F, $\mathbb{1}_V \star F$, $(\mathbb{1}_V \star F)' = \mathbb{1}'_V \star F$



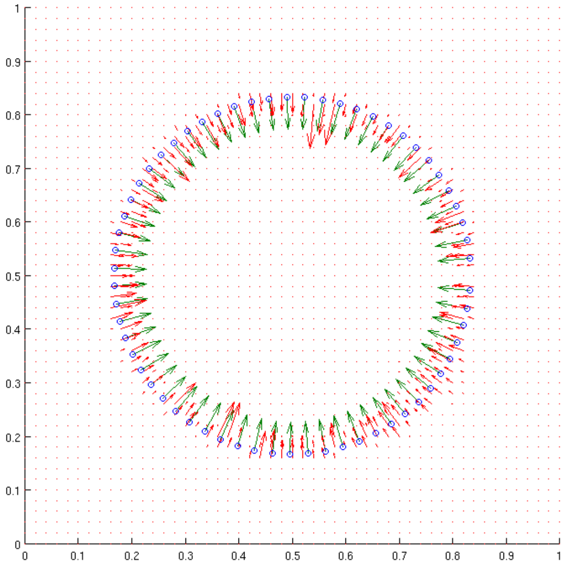
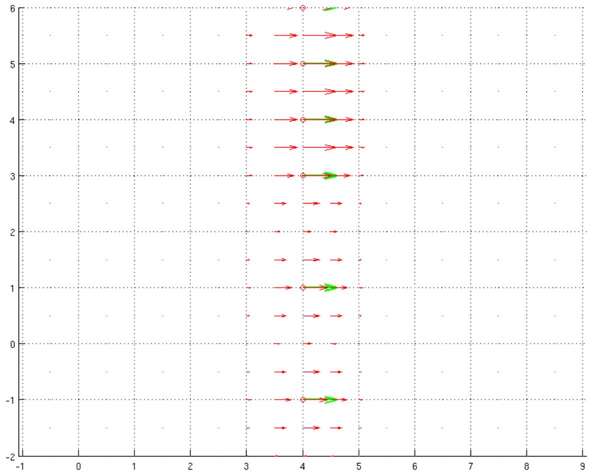
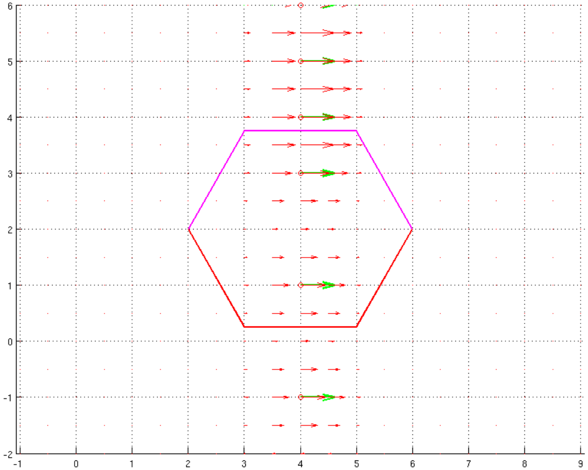


FIGURE: $\vec{V}$ interpolé à partir d'un cercle.

En dimension supérieure à 1, problème :



En dimension supérieure à 1, problème : $\int \vec{V} \cdot \mathrm{d}\vec{l} \neq \int \vec{V} \cdot \mathrm{d}\vec{l}$

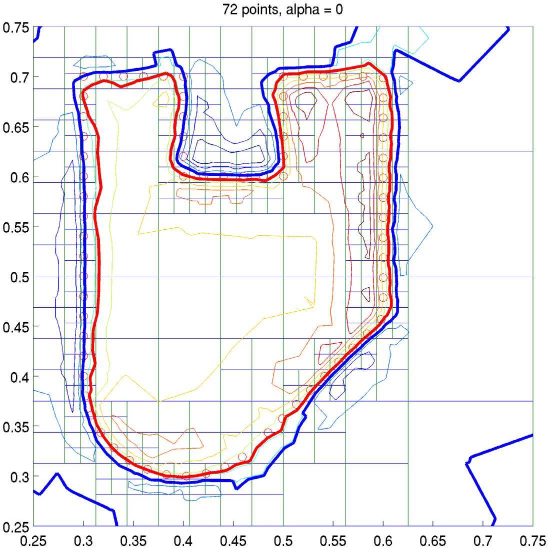


On cherche χ qui minimise

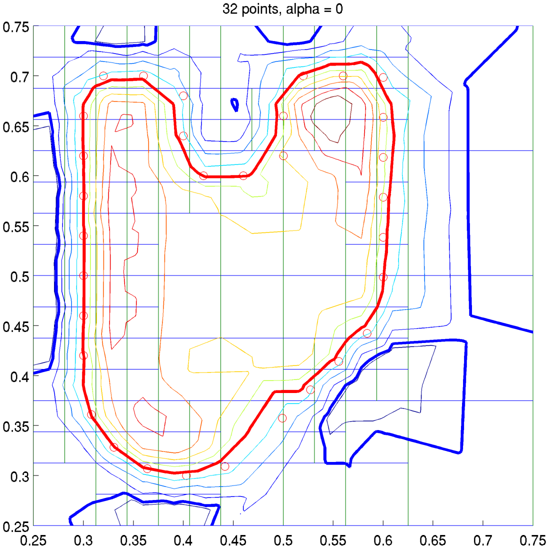
$$E_{\vec{V}}(\chi) = \int \left\| \nabla\chi(p) - \vec{V}(p) \right\|^2 \mathrm{d}p. \tag{1}$$

C'est la méthode de Poisson, simple, globale et robuste.

La fonction χ dérive : on prend $\{\chi \geqslant \frac{1}{|P|} \sum_{p \in P} \chi(p)\}$ et non $\{\chi \geqslant 1/2\}$



Même ainsi, la reconstruction est trop lisse.



On souhaite ajouter un terme d'attache aux données – en remplaçant χ par $\chi - 1/2$:

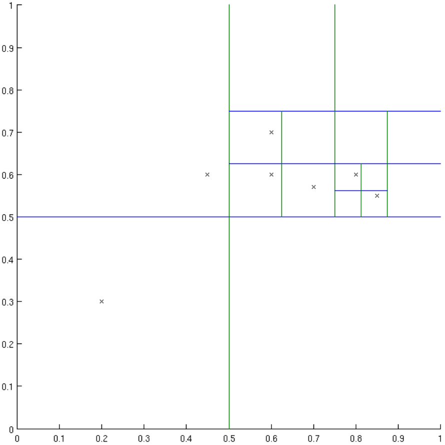
$$E_P(\chi) = \frac{\mathrm{Aire}(P)}{|P|} \sum_{p \in P} \chi^2(p) \tag{2}$$

On minimise alors

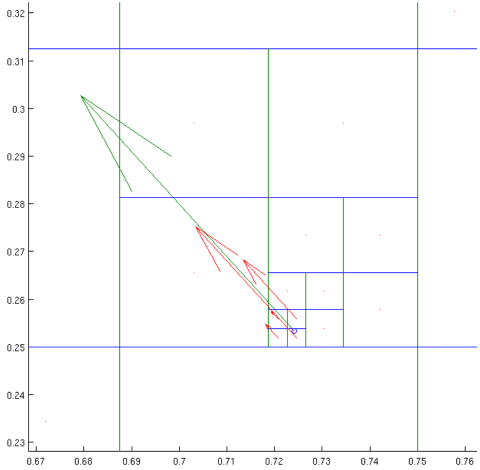
$$E(\chi) = E_{\vec{V}}(\chi) + \alpha E_P(\chi) \tag{3}$$

Détails de l'implémentation

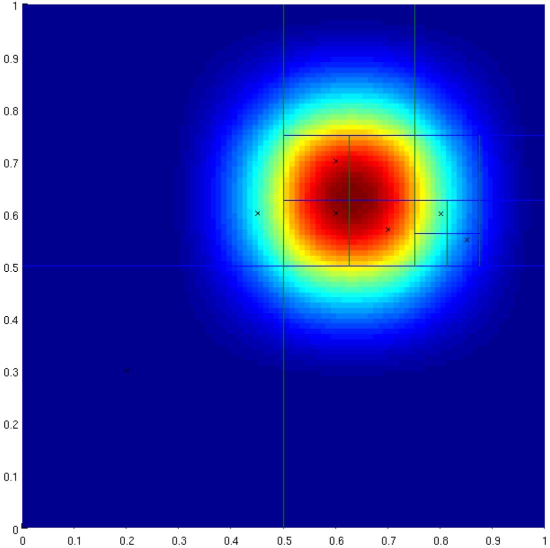
Utilisation d'un Quadtree - Octree



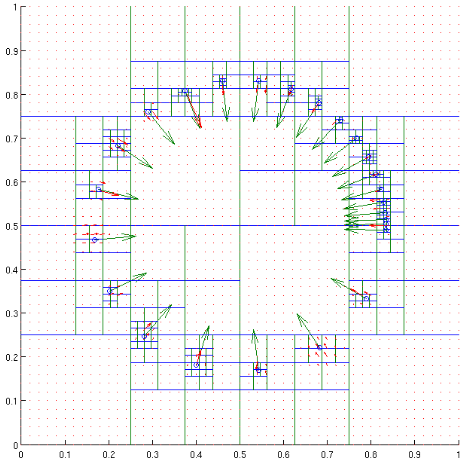
Pour simplifier les calculs, on “diffuse” un point sur ses 4 voisins.



On utilise les fonction nodales F_n

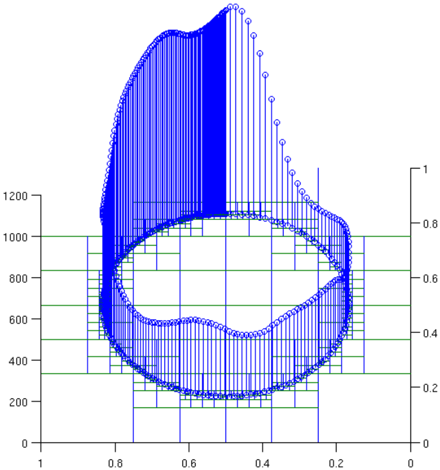


Une diffusion uniforme donne de mauvais résultats.

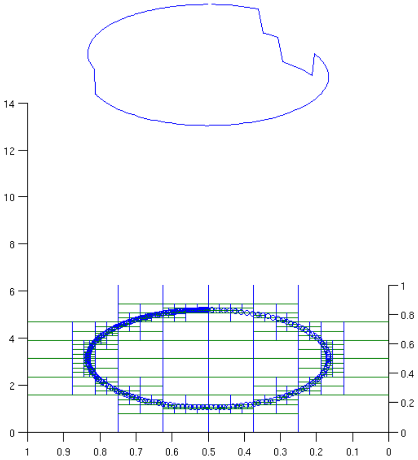


$$W_{\tilde{D}}(q) = \sum_{p \in P} \sum_{n \in \mathrm{Vois}_{\tilde{D}}(p)} \alpha_{n,p} F_n(q) \tag{4}$$

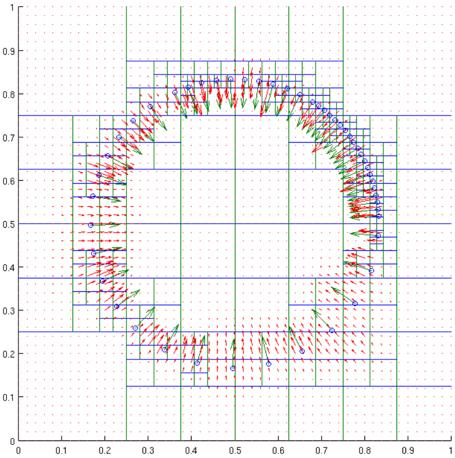
$$\simeq \sum_{p \in P} F_{2^{-\tilde{D}}}(q - p), \tag{5}$$



$$\mathrm{Prof}(p) = \min(D, D + \log_4(W_{\tilde{D}}(p)/\overline{W_{\tilde{D}}})) \tag{6}$$



$$\vec{V}(q) = \sum_{p \in P} \frac{1}{W_{\tilde{D}}(p)} \sum_{n \in \mathrm{Vois}_{\mathrm{Prof}(p)}} \alpha_{n,p} F_n(q) \vec{N}_p \tag{7}$$



Calcul de χ par la méthode des éléments finis

$$E(\chi) = \langle \vec{V} - \nabla\chi, \vec{V} - \nabla\chi \rangle_{[0,1]^3} + \alpha\langle \chi, \chi \rangle_P \tag{8}$$

avec

$$\langle \vec{U}, \vec{V} \rangle_{[0,1]^3} = \iiint \langle \vec{U}(x), \vec{V}(x) \rangle \mathrm{d}x, \tag{9}$$

$$\langle F, G \rangle_P = \frac{\mathrm{Aire}(P)}{|P|} \sum_{p \in P} F(p)G(p). \tag{10}$$

On cherche χ sous la forme

$$\chi(p) = \sum_{n \in T} \lambda_n F_n(p) \tag{11}$$

$$\begin{aligned} E(\lambda) &= \langle \vec{V} - \nabla \sum \lambda_n F_n, \vec{V} - \nabla \sum \lambda_n F_n \rangle_{[0,1]^3} && (12) \\ &+ \alpha \cdot \langle \sum \lambda_n F_n, \sum \lambda_n F_n \rangle_P && (13) \\ &= \lambda^T A \lambda - 2b^T \lambda + \langle \vec{V}, \vec{V} \rangle && (14) \end{aligned}$$

avec

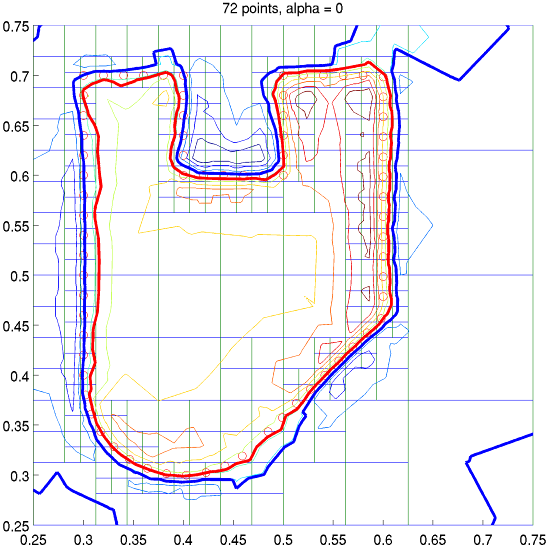
$$\begin{aligned} A_{i,j} &= \langle \nabla F_i, \nabla F_j \rangle + \alpha \cdot \langle F_i, F_j \rangle_P, && (15) \\ b_i &= \langle \vec{V}, \nabla F_i \rangle. && (16) \end{aligned}$$

À l'optimum, on a

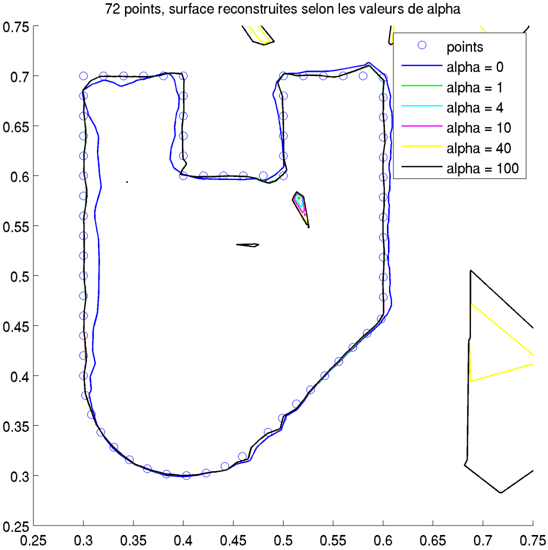
$$A\lambda = b \tag{17}$$

Résultats

Défaut de la méthode de Poisson : il se peut qu'aucune ligne ne soit pertinente, à cause de la dérive de χ



La méthode écrantée permet, elle, de résoudre ce problème



Tout en conservant une robustesse similaire à celle de la méthode de Poisson